

Updated QCD Global Analysis of SSAs: $\tilde{H}(z)$, the Soffer Bound, and Lattice g_T



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Transverse Polarization Phenomenon

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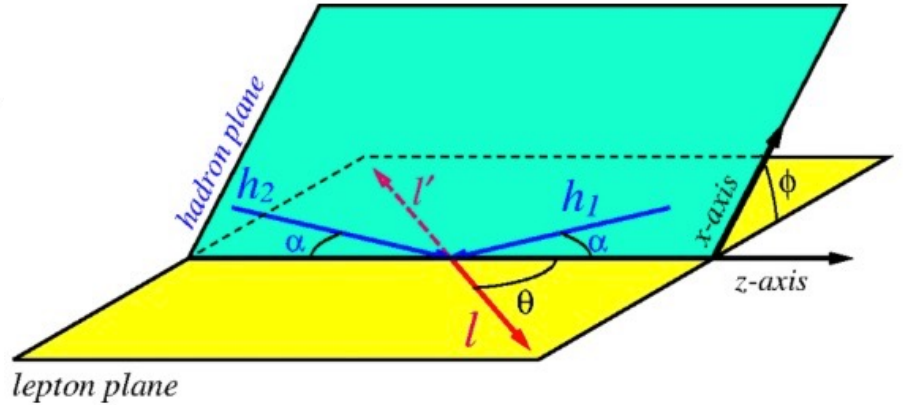
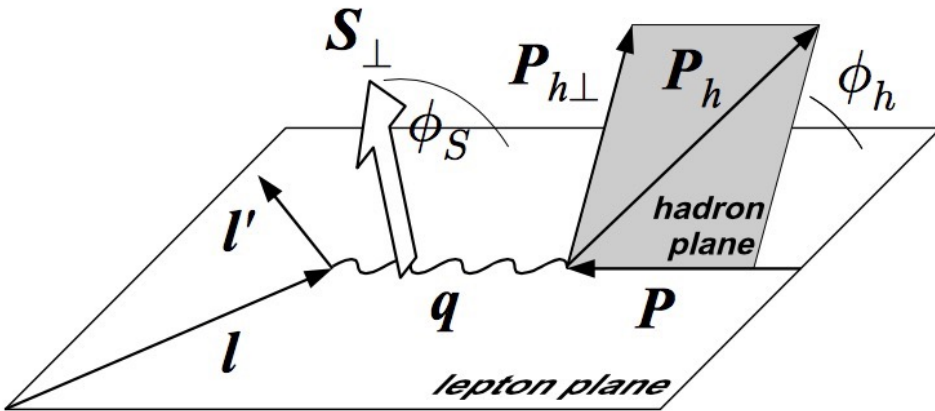
May 24, 2022



Background

$$lp^\uparrow \rightarrow lhX$$

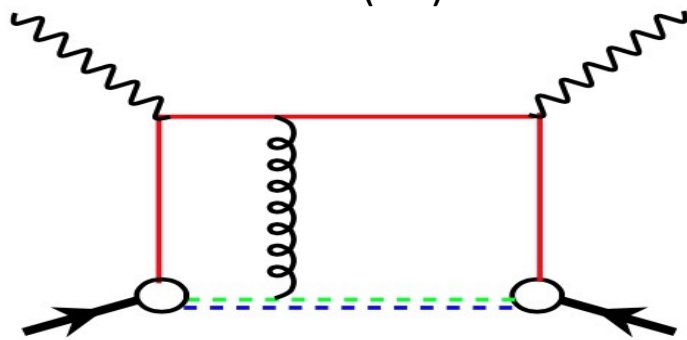
$$\{\pi, p\}p^\uparrow \rightarrow \{l^+l^-, W^\pm, Z\}X$$



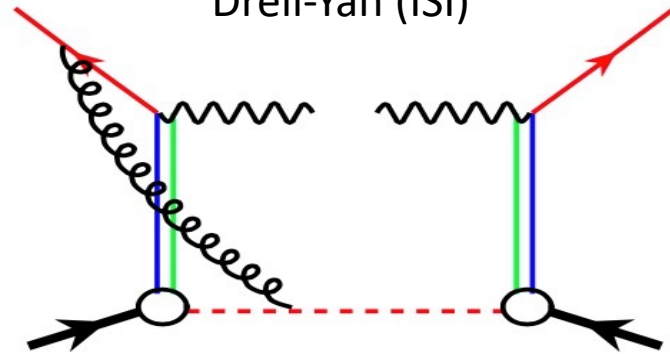
$$F_{UT}^{\sin(\phi_h - \phi_S)} = C \left[-\frac{\hat{h} \cdot \vec{k}_T}{M} f_{1T}^\perp D_1 \right]$$

$$F_{TU}^{\sin \phi} = C \left[-\frac{\hat{h} \cdot \vec{k}_{aT}}{M_a} f_{1T}^\perp \bar{f}_1 \right]$$

SIDIS (FSI)



Drell-Yan (ISI)

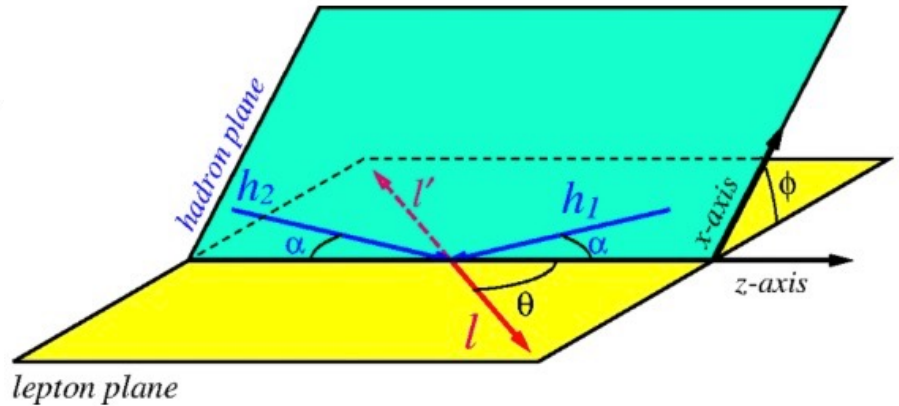
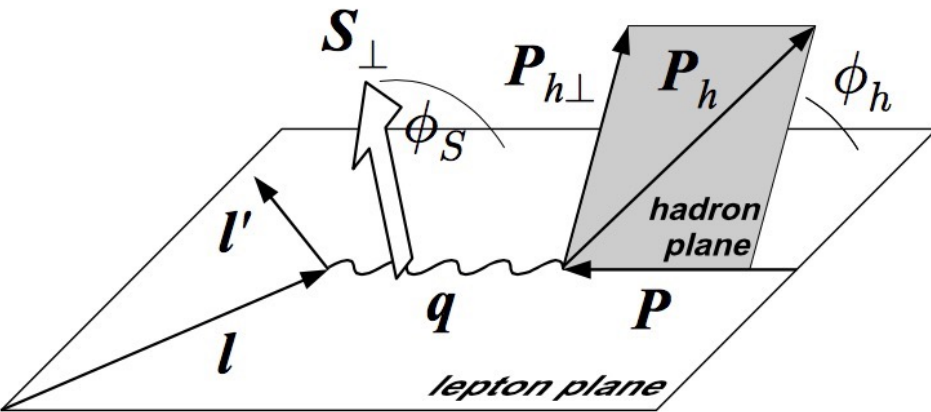


$$f_{1T}^\perp(x, \vec{k}_T^2)|_{SIDIS} = -f_{1T}^\perp(x, \vec{k}_T^2)|_{DY}$$

(Brody, Hwang, Schmidt (2002), Collins (2002))

$$lp^\uparrow \rightarrow lhX$$

$$\{\pi, p\}p^\uparrow \rightarrow \{\ell^+ \ell^-, W^\pm, Z\}X$$



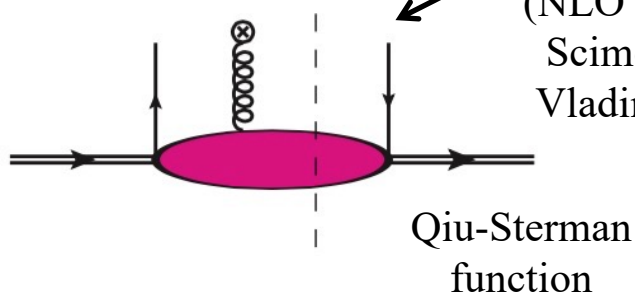
$$F_{UT}^{\sin(\phi_h - \phi_S)} = C \left[-\frac{\hat{h} \cdot \vec{k}_T}{M} f_{1T}^\perp D_1 \right]$$

$$F_{TU}^{\sin \phi} = C \left[-\frac{\hat{h} \cdot \vec{k}_{aT}}{M_a} f_{1T}^\perp \bar{f}_1 \right]$$

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim F_{FT}(x, x; \mu_{b_*}) \exp \left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q) \right]$$

OPE
(NLO available from
Scimemi, Tarasov,
Vladimirov (2019))

$g_{f_{1T}^\perp}(x, b_T) + g_K(b_T) \ln(Q/Q_0)$
(Aybat, et al. (2012); Echevarria, et al. (2014))

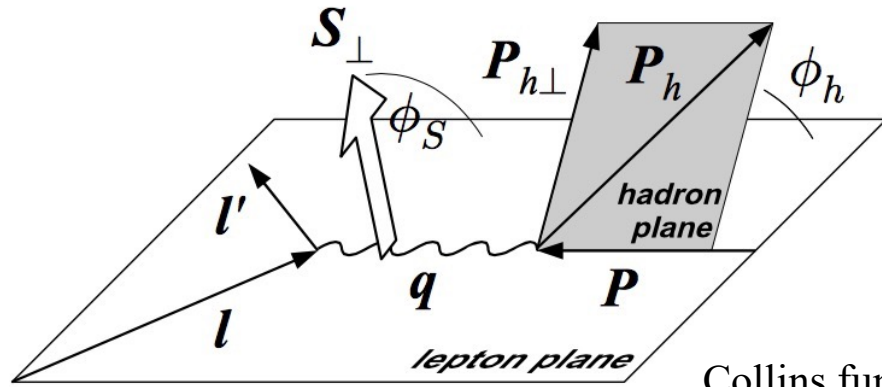


Parton model

$$\pi F_{FT}(x, x) = \int d^2 \vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^\perp(x, k_T^2) \equiv f_{1T}^{\perp(1)}(x)$$

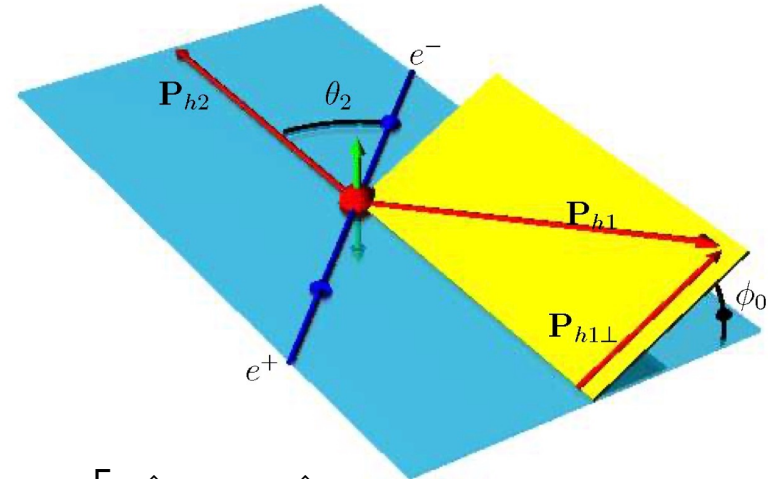
(Boer, Mulders, Pijlman (2003)) 1

$$lp^\uparrow \rightarrow lhX$$



Collins function

$$e^+e^- \rightarrow h_1 h_2 X$$

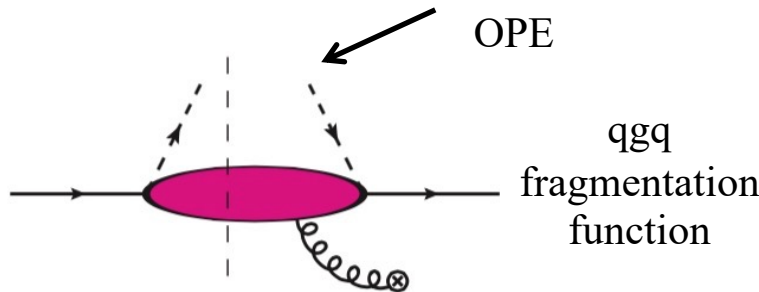


$$F_{UT}^{\sin(\phi_h + \phi_S)} = C \left[-\frac{\hat{h} \cdot \vec{p}_\perp}{M_h} h_1 H_1^\perp \right] \quad F_{UU}^{\cos(2\phi_0)} = C \left[\frac{2\hat{h} \cdot \vec{p}_{a\perp} \hat{h} \cdot \vec{p}_{b\perp} - \vec{p}_{a\perp} \cdot \vec{p}_{b\perp}}{M_a M_b} H_1^\perp \bar{H}_1^\perp \right]$$

$$\tilde{h}_1(x, b_T; Q^2, \mu_Q) \sim h_1(x; \mu_{b_*}) \exp[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{h_1}(b_T, Q)]$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim H_1^{\perp(1)}(z; \mu_{b_*}) \exp[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)]$$

(Kang, et al. (2016))

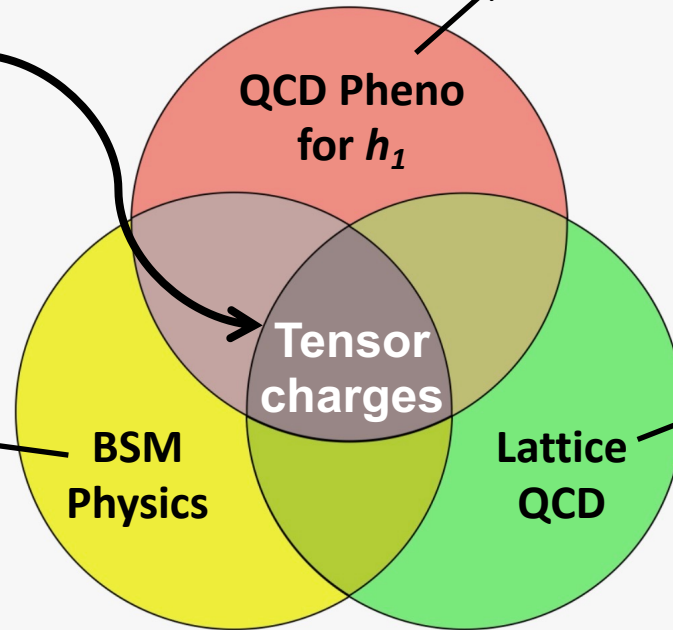


Parton model

$$H_1^{\perp(1)}(z) = z^2 \int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} H_1^\perp(z, z^2 p_\perp^2)$$

$$\delta q \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)] \quad g_T \equiv \delta u - \delta d$$

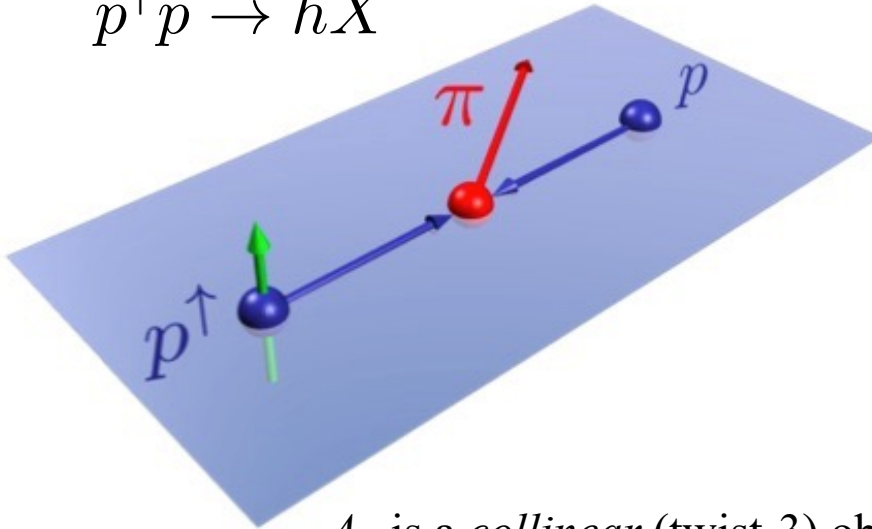
Anselmino, et al. (2013, 2015); Goldstein, et al. (2014);
 Radici, et al. (2013, 2018); Kang, et al. (2016);
 Benel, et al. (2019); D'Alesio, et al. (2020);
 Cammarota, et al. (2020)



Courtoy, et al. (2015);
 Yamanaka, et al. (2017);
 Liu, et al. (2018);
 Gonzalez-Alonso, et al. (2019)

Gupta, et al. (2018);
 Yamanaka, et al. (2018);
 Hasan, et al. (2019);
 Alexandrou, et al. (2019)

$$p^\uparrow p \rightarrow hX$$



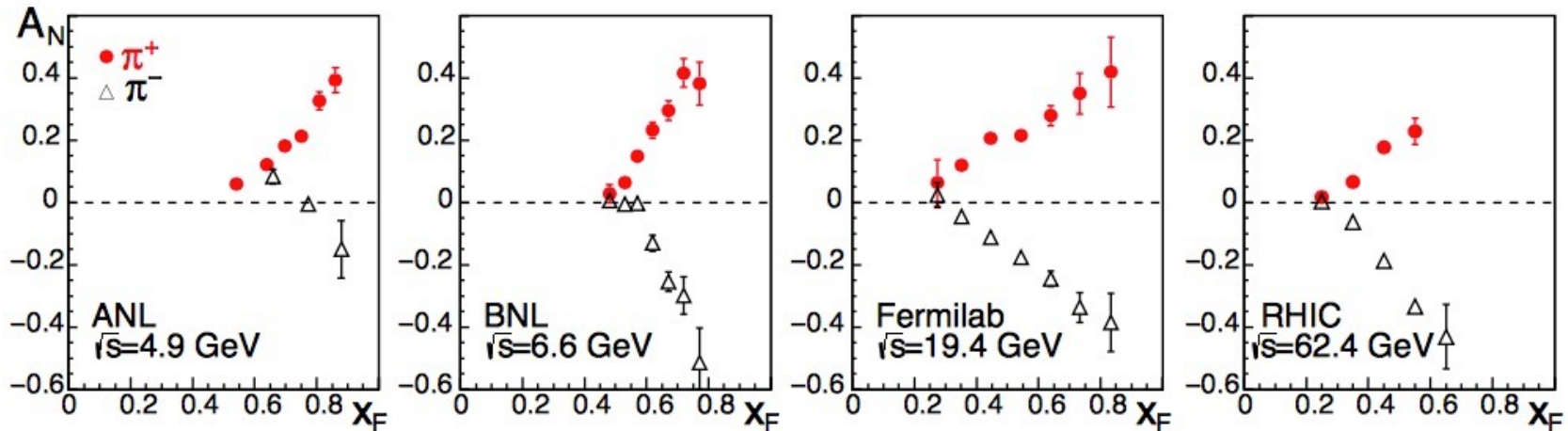
$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \mathbf{F}_{FT} \otimes D_1}_{\text{Qiu-Sterman term}}$$

Qiu-Sterman term

$$+ \underbrace{H_F \otimes f_1 \otimes \mathbf{h}_1 \otimes \left(H_1^{\perp(1)}, \tilde{H} \right)}_{\text{Fragmentation term}}$$

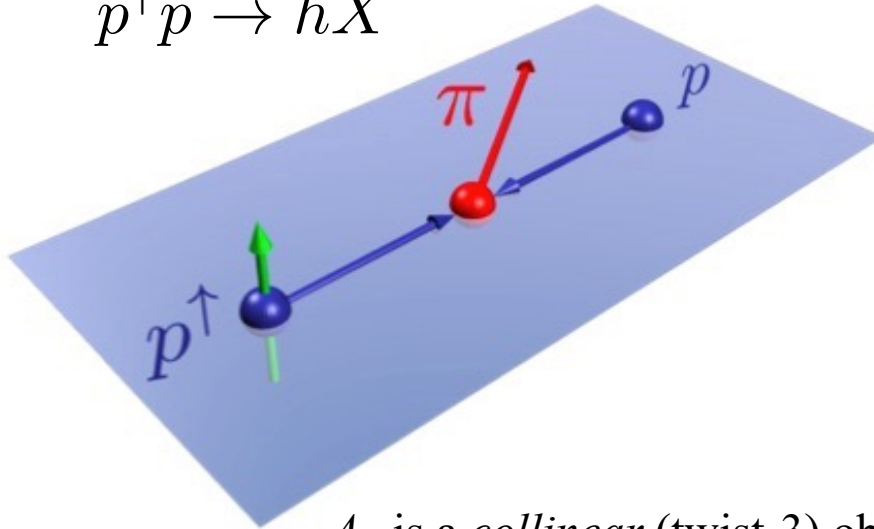
Fragmentation term

A_N is a *collinear* (twist-3) observable



1976 →

$$p^\uparrow p \rightarrow hX$$

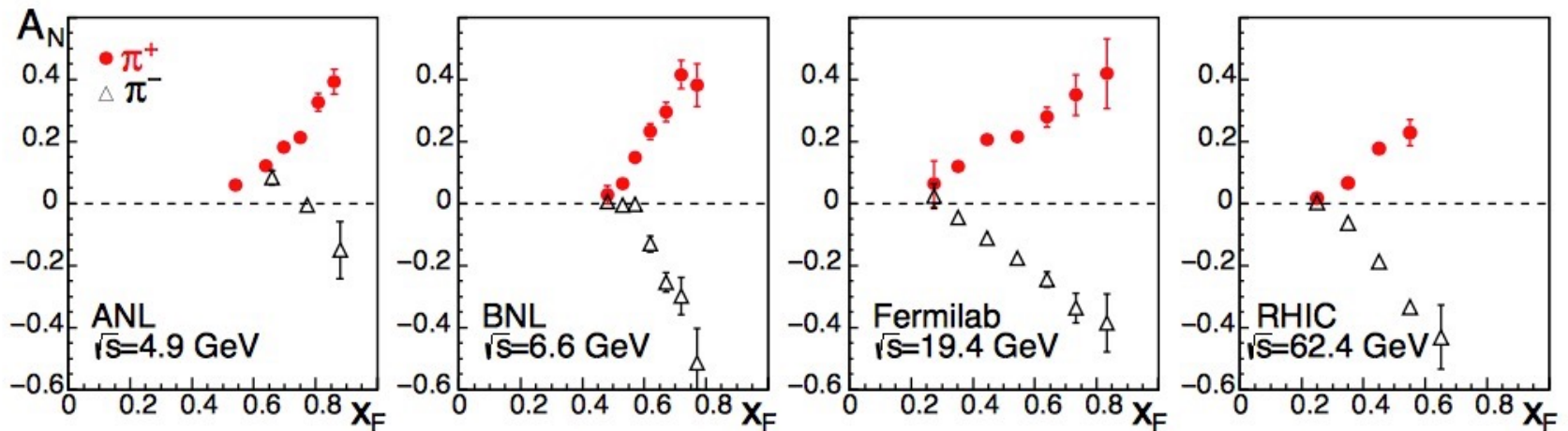


$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes F_{FT} \otimes D_1}_{\text{Qiu-Sterman term}}$$

$$+ \underbrace{H_F \otimes f_1 \otimes h_1 \otimes \left(H_1^{\perp(1)}, \tilde{H} \right)}_{\text{Fragmentation term}}$$

(Metz, DP (2012); Kanazawa, et al. (2014);
Gamberg, et al. (2017); Cammarota, et al. (2020))

A_N is a *collinear* (twist-3) observable



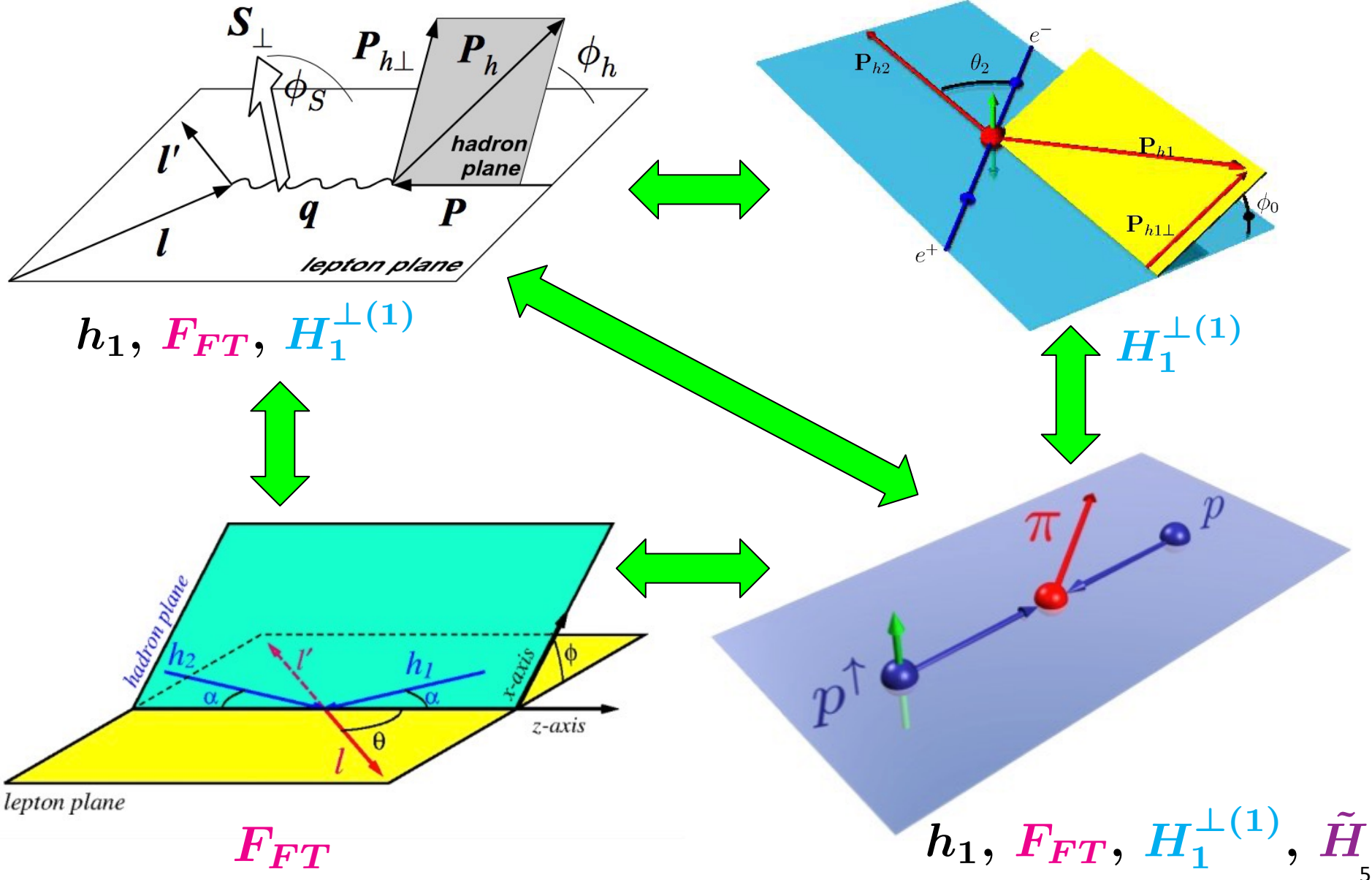
1976 →



Simultaneous QCD Global Analysis of SSAs



Cammarota, Gamberg, Kang, Miller, DP, Prokudin, Rogers, Sato, PRD 102 (2020)



- In 2020 we performed the first global analysis of SSAs in SIDIS, Drell-Yan, e^+e^- annihilation, and proton-proton collisions and extracted a universal set of non-perturbative functions

$$h_1(x), F_{FT}(x, x), H_1^{\perp(1)}(z), \hat{H}(z) \quad \text{noise in the fit – need } A_{UT}^{\sin \phi_S}$$

along with the relevant transverse momentum widths for the Sivers, transversity, and Collins functions: $\langle k_T^2 \rangle_{f_{1T}^\perp}, \langle k_T^2 \rangle_{h_1}, \langle p_\perp^2 \rangle_{H_1^\perp}^{fav}, \langle p_\perp^2 \rangle_{H_1^\perp}^{unf}$

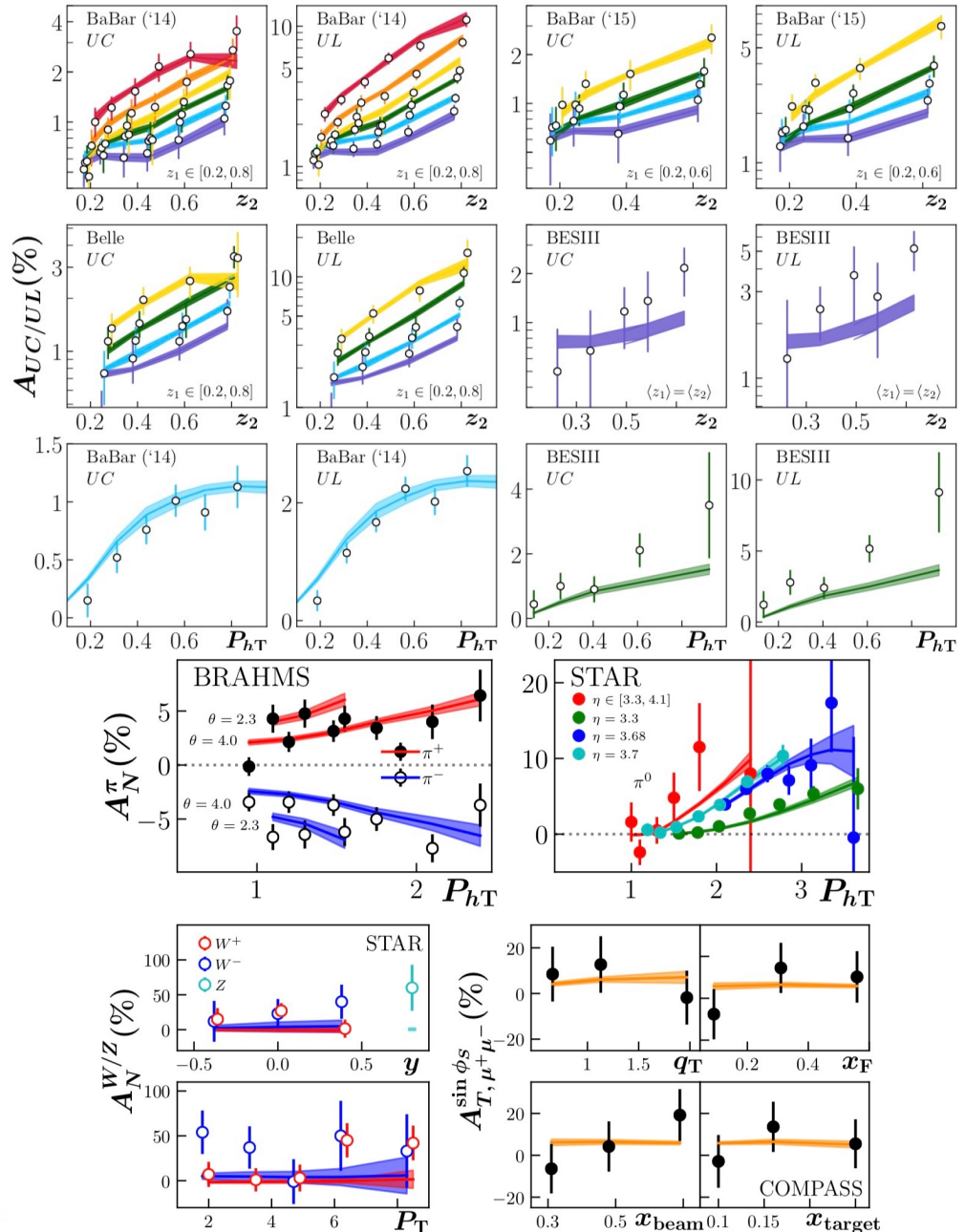
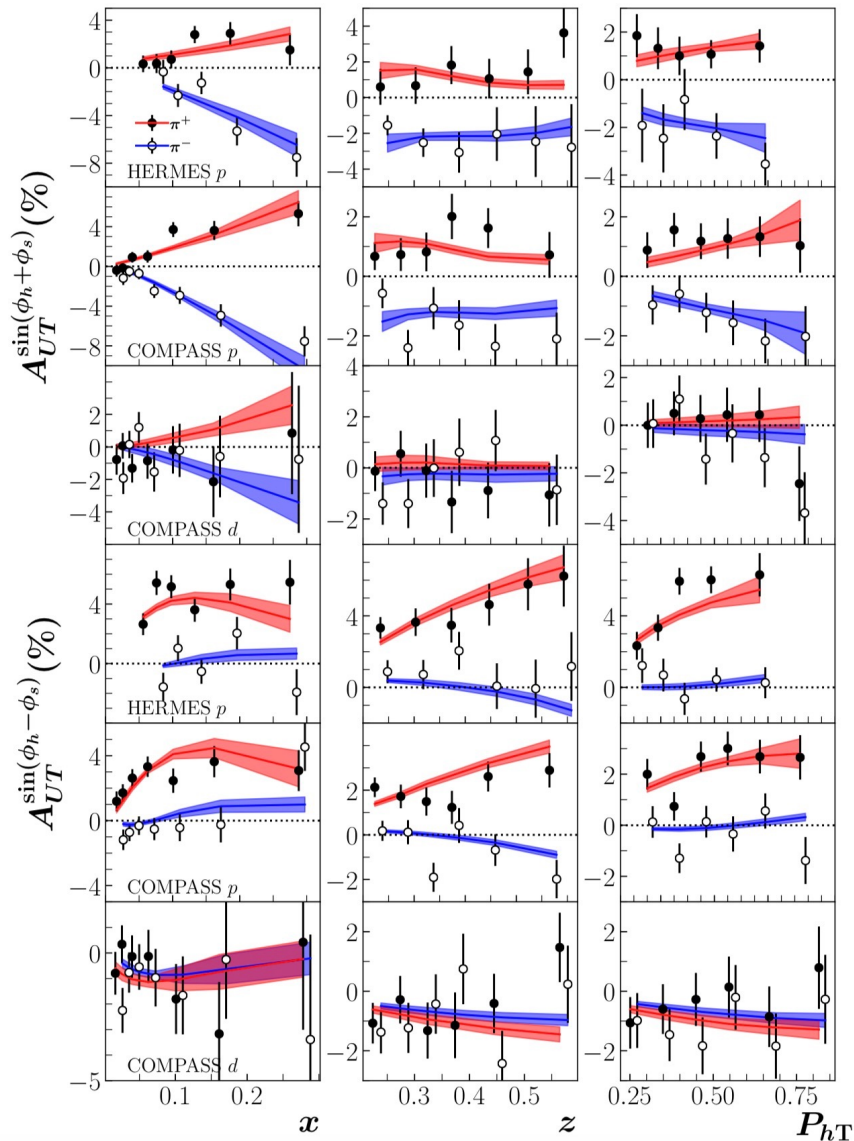
- We use a Gaussian ansatz: $F(x, k_T^2) \sim F(x)e^{-k_T^2/\langle k_T^2 \rangle}$ where

$$F^q(x) = \frac{N_q x^{a_q} (1-x)^{b_q} (1 + \gamma_q x^{\alpha_q} (1-x)^{\beta_q})}{B[a_q + 2, b_q + 1] + \gamma_q B[a_q + \alpha_q + 2, b_q + \beta_q + 1]}$$

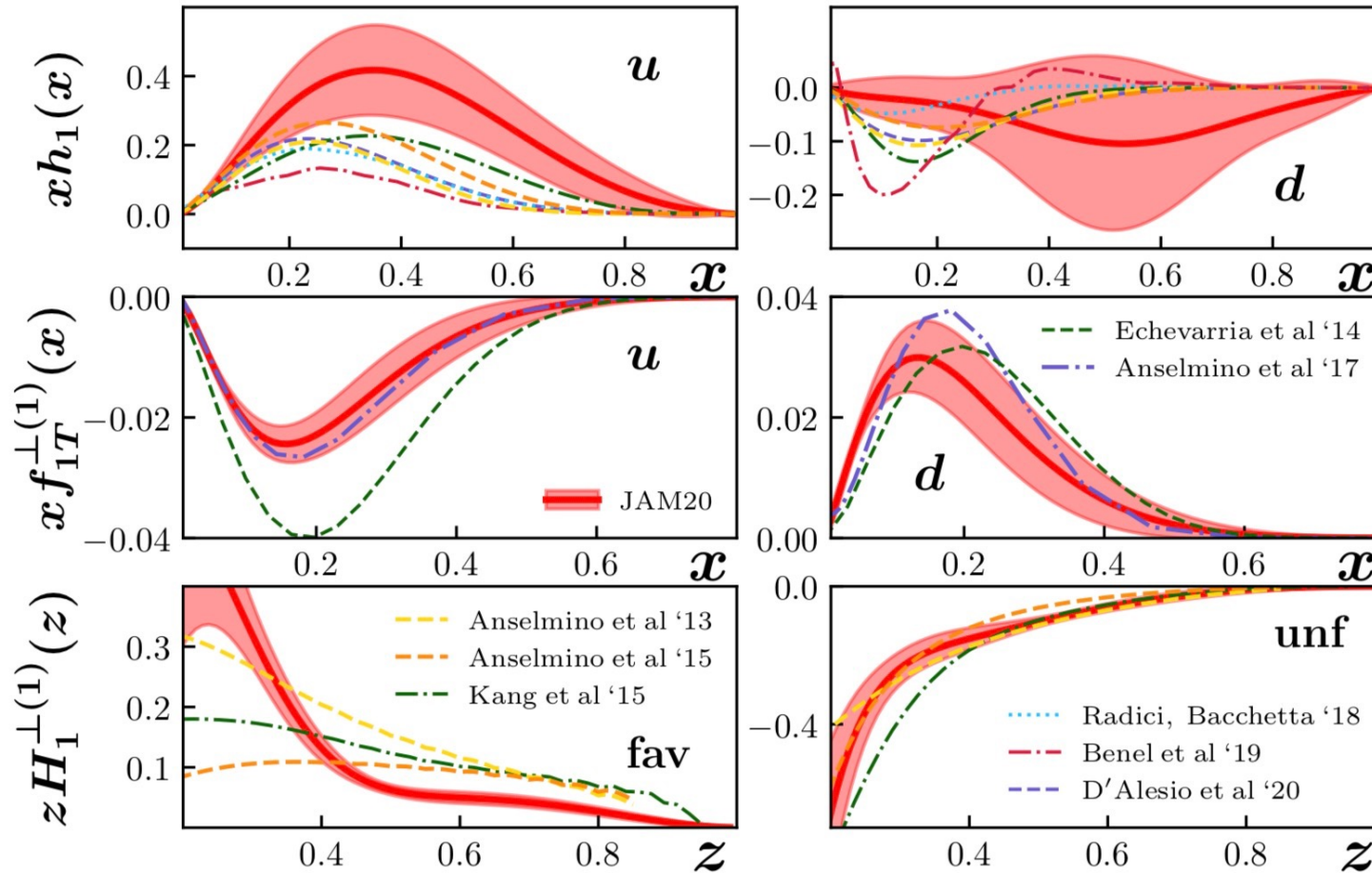
NB: $\{\gamma, \alpha, \beta\}$ only used for Collins function

- DGLAP-type evolution for the collinear functions analogous to Duke & Owens (1984): double-log Q^2 -dependent term explicitly added to the parameters

$$\chi^2 / N_{\text{pts.}} = 520 / 517 = 1.01$$



JAM3D-20

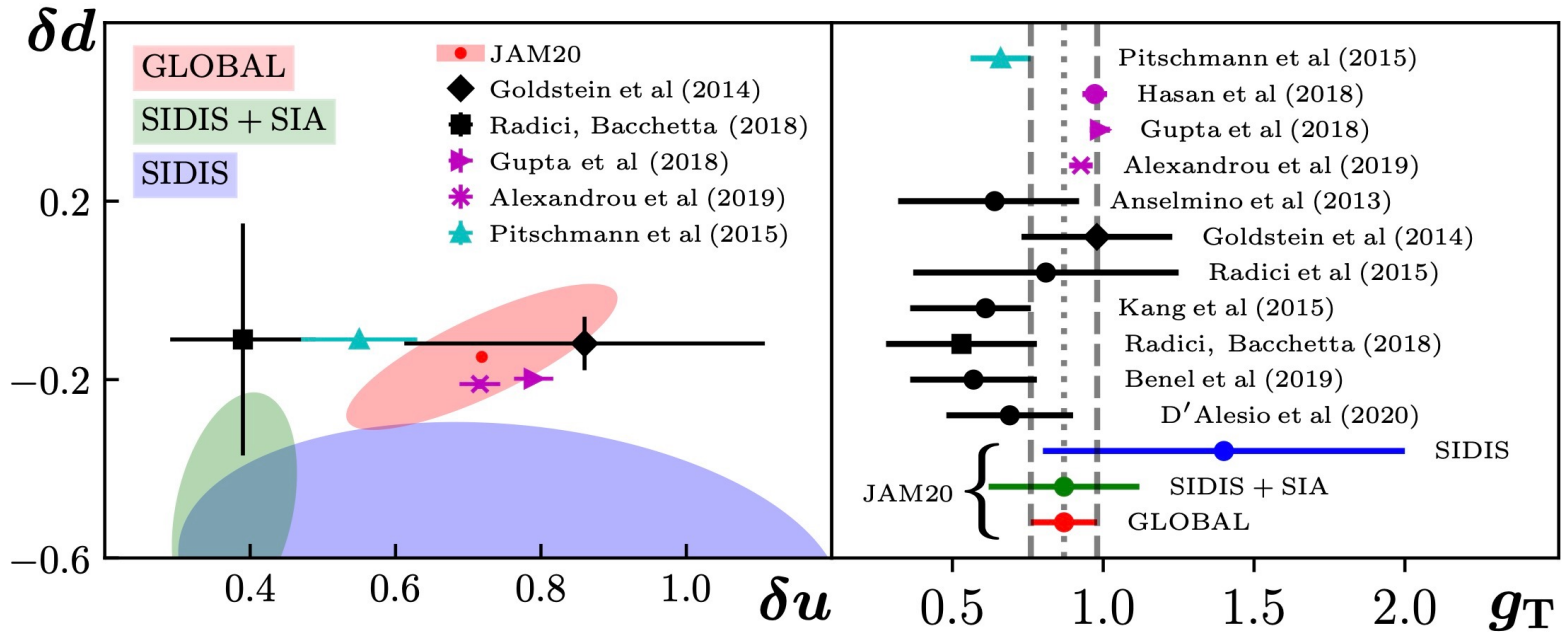


Transversity

Sivers first moment (QS function)

Collins first moment

Cammarota, Gamberg, Kang, Miller, DP, Prokudin, Rogers, Sato, PRD 102 (2020)



Cammarota, Gamberg, Kang, Miller, DP, Prokudin, Rogers, Sato, PRD **102** (2020)

Only after a *simultaneous* QCD global analysis of SSAs does the phenomenological extraction of the tensor charges agree with lattice.



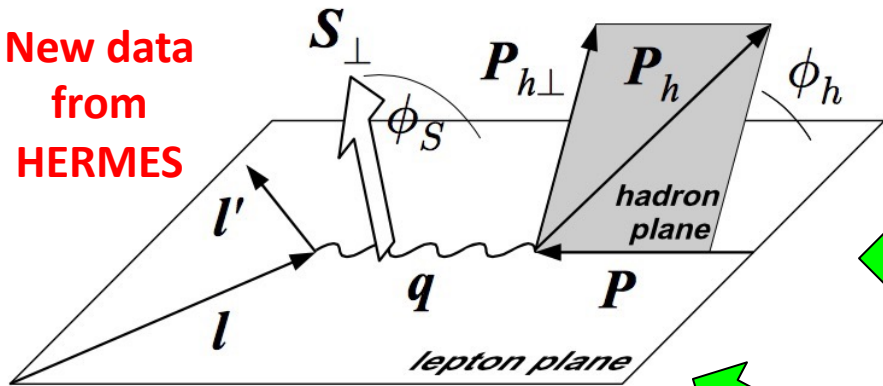
Updated QCD Global Analysis of SSAs

Gamberg, Malda, Miller, DP, Prokudin, Sato, arXiv:2205.00999 [hep-ph], submitted to PRD

User-friendly jupyter notebook to calculate functions and asymmetries:

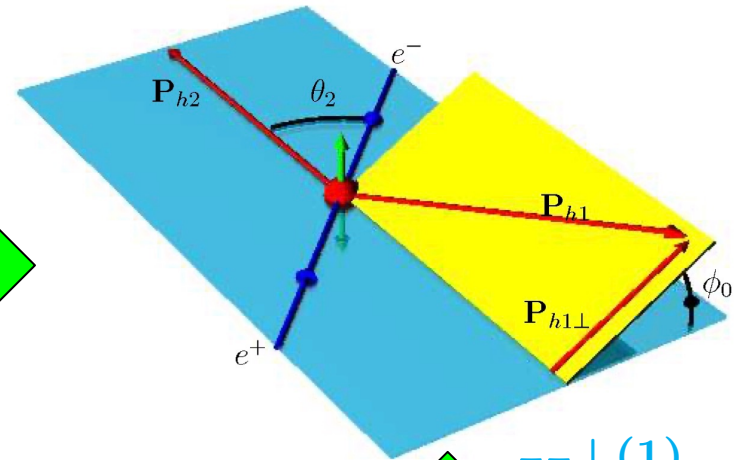
https://colab.research.google.com/github/pitonyak25/jam3d_dev_lib/blob/main/JAM3D_Library.ipynb

New data from HERMES

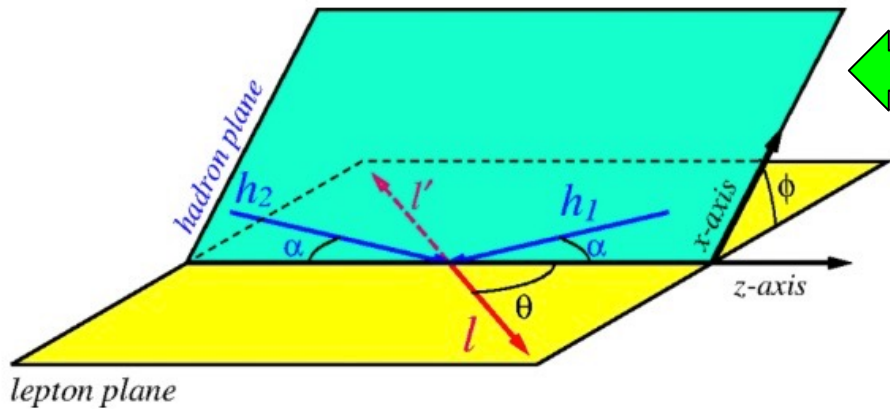


$h_1, F_{FT}, H_1^{\perp(1)}, \tilde{H}$

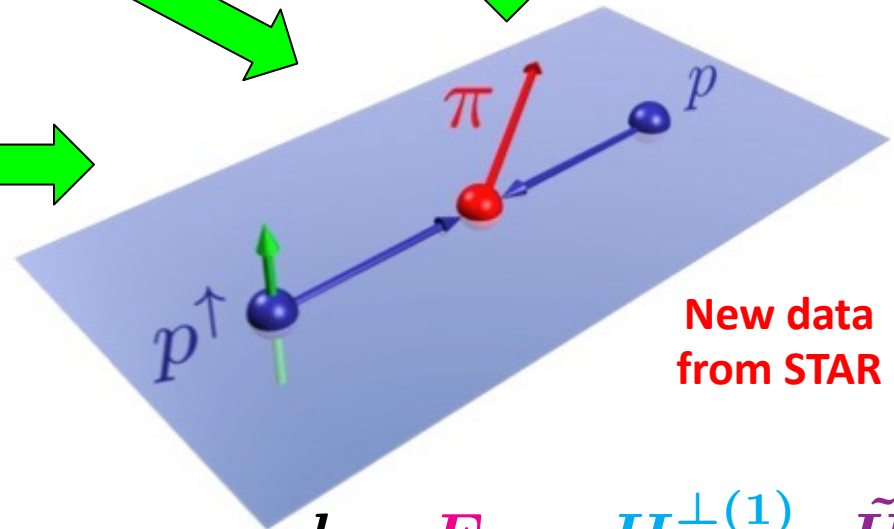
$A_{UT}^{\sin \phi_S}$ now available



$H_1^{\perp(1)}$



F_{FT}



New data from STAR

$h_1, F_{FT}, H_1^{\perp(1)}, \tilde{H}$

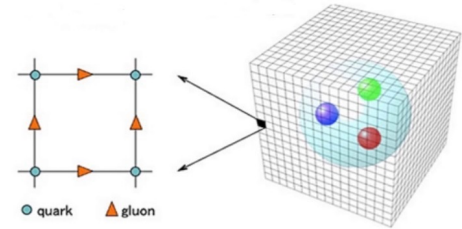
➤ Additional data/constraints included in the fit:

- Collins and Sivers effects (3D-binned) SIDIS data from HERMES (2020)
- $A_{UT}^{\sin \phi_S}$ data (x and z projections only) from HERMES (2020)



$$\int d^2\vec{P}_{hT} F_{UT}^{\sin \phi_S} = -\frac{x}{z} \sum_q e_q^2 \frac{2M_h}{Q} h_1^{q/N}(x) \tilde{H}^{h/q}(z)$$

- Lattice data on g_T at the physical pion mass from Alexandrou, et al. (2019)



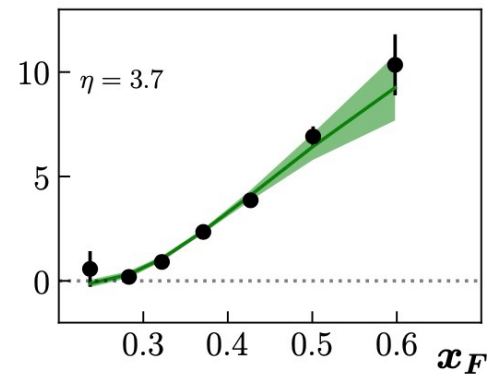
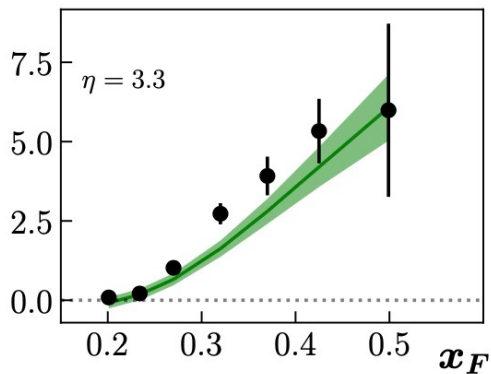
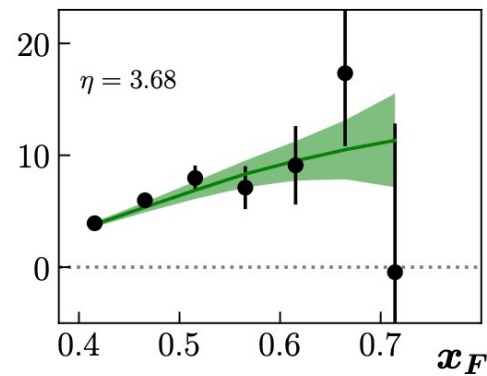
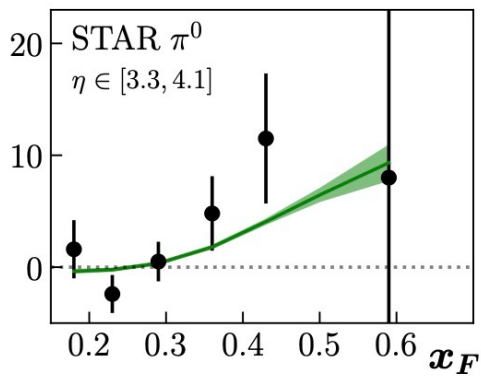
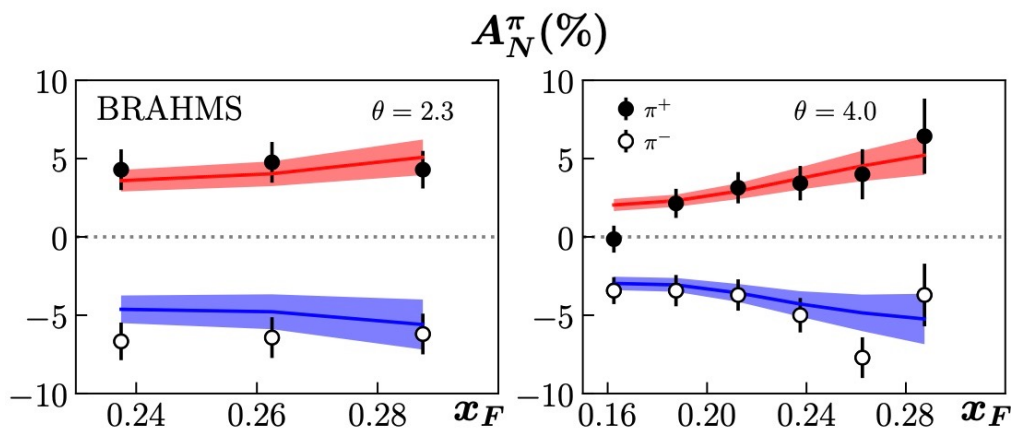
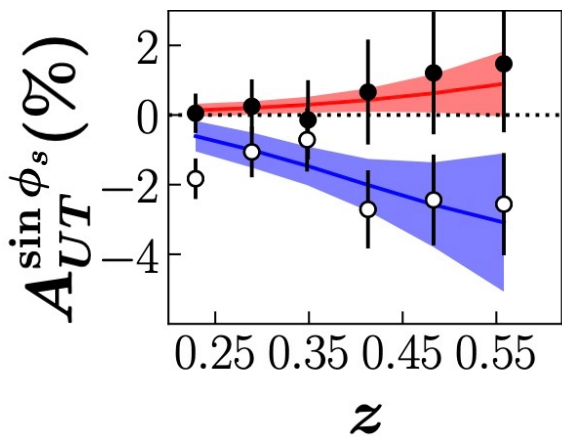
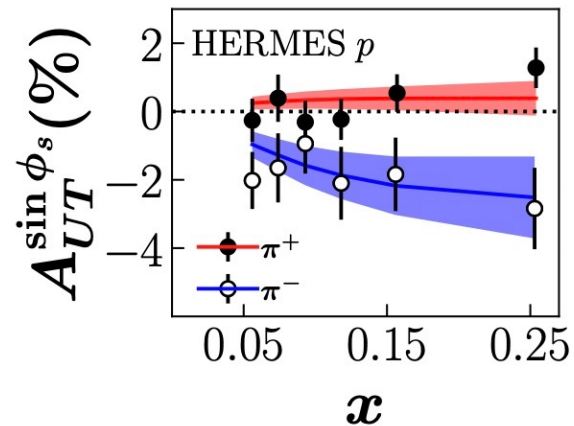
- Imposing the Soffer bound on transversity: $|h_1^q(x)| \leq \frac{1}{2}(f_1^q(x) + g_1^q(x))$

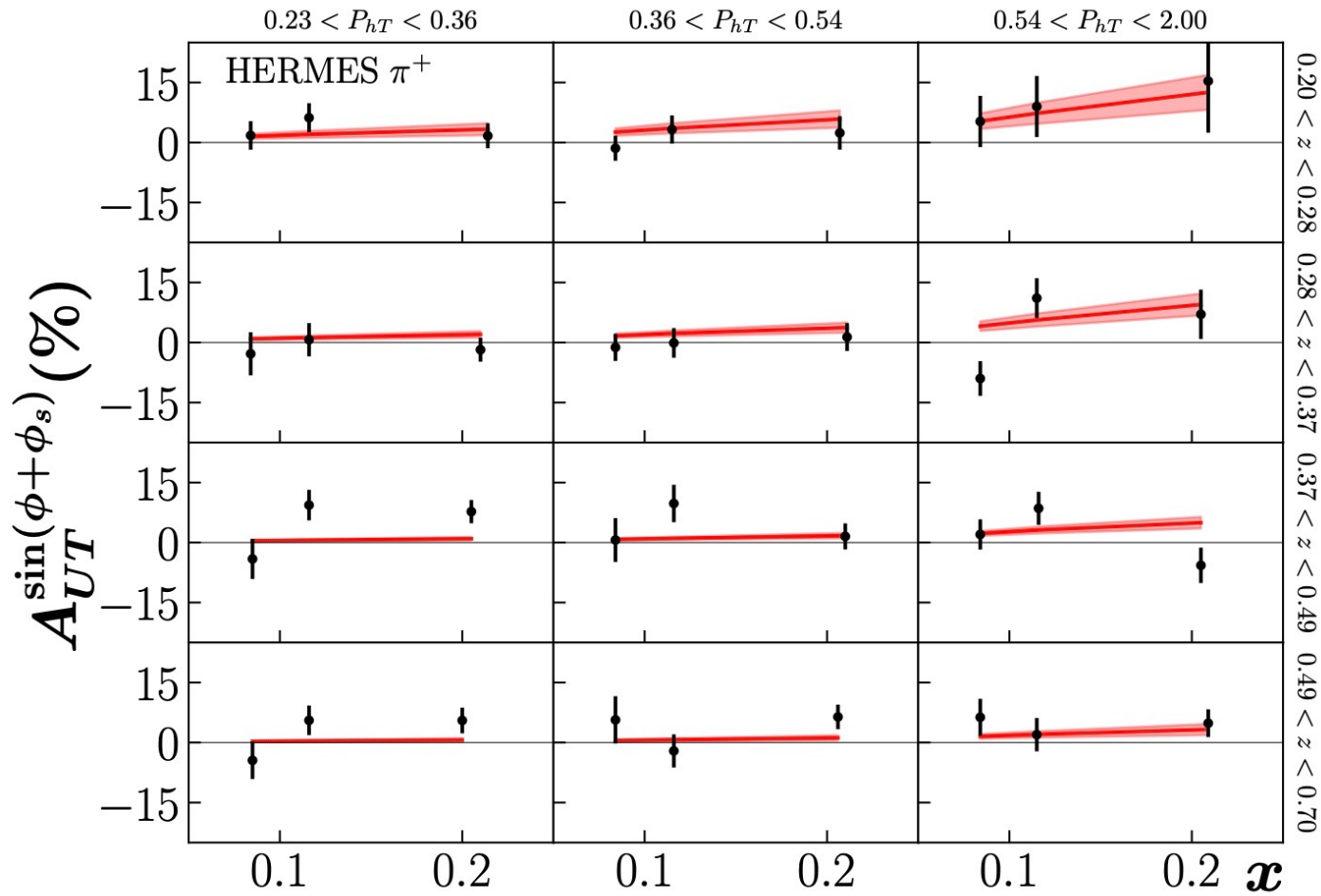
Generate “data” (central value and 1- σ uncertainty) using recent simultaneous fit of f_1 and g_1 from Cocuzza, et al. (2022) and add to the χ^2 if SB is violated by more than the uncertainty in the data

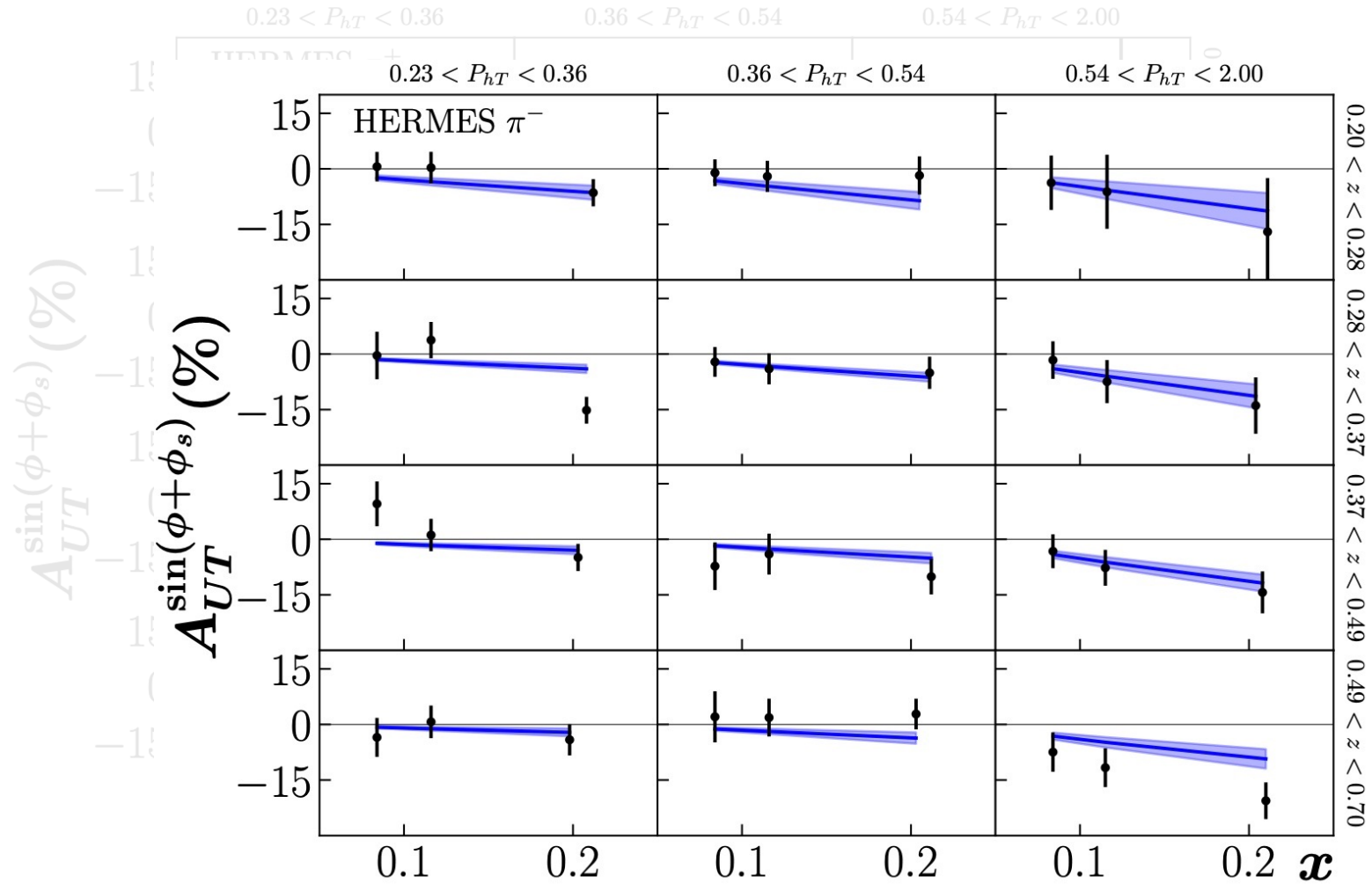
$$\chi^2/N_{\text{pts.}} = 647/634 = 1.02$$

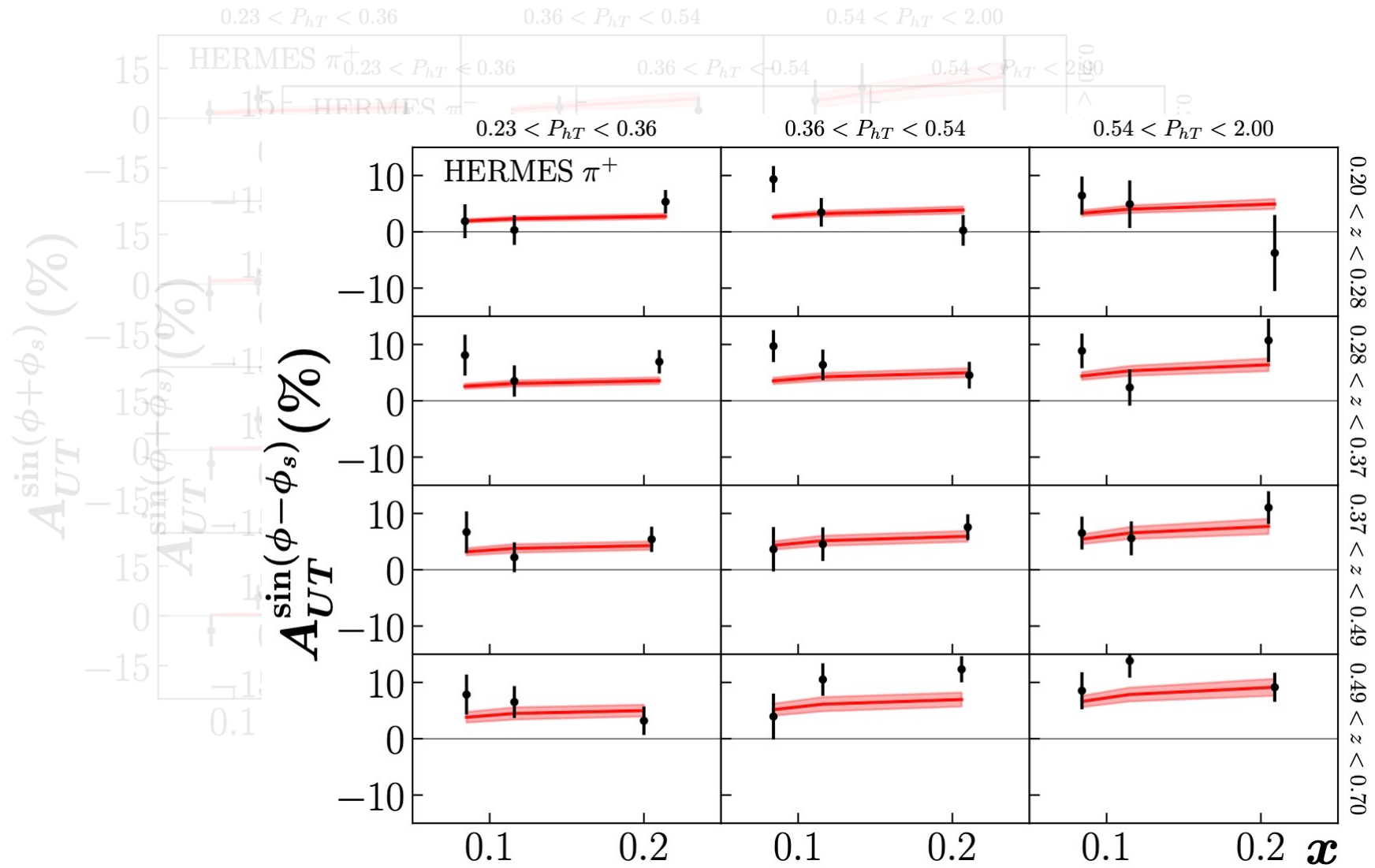
Observable	Reactions	Non-Perturbative Function(s)	χ^2/npts
$A_{UT}^{\sin(\phi_h - \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	$182.9/166 = 1.10$
$A_{UT}^{\sin(\phi_h + \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x, \vec{k}_T^2), H_1^\perp(z, z^2 \vec{p}_T^2)$	$181.0/166 = 1.09$
$*A_{UT}^{\sin \phi_S}$	$e + p^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), \tilde{H}(z)$	$18.6/36 = 0.52$
$A_{UC/UL}$	$e^+ + e^- \rightarrow \pi^+ \pi^- (UC, UL) + X$	$H_1^\perp(z, z^2 \vec{p}_T^2)$	$154.9/176 = 0.88$
$A_{T, \mu^+ \mu^-}^{\sin \phi_S}$	$\pi^- + p^\uparrow \rightarrow \mu^+ \mu^- + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	$6.92/12 = 0.58$
$A_N^{W/Z}$	$p^\uparrow + p \rightarrow (W^+, W^-, Z) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	$30.8/17 = 1.81$
A_N^π	$p^\uparrow + p \rightarrow (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), F_{FT}(x, x) = \frac{1}{\pi} f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z), \tilde{H}(z)$	$70.4/60 = 1.17$
Lattice g_T	—	$h_1(x)$	$1.82/1 = 1.82$

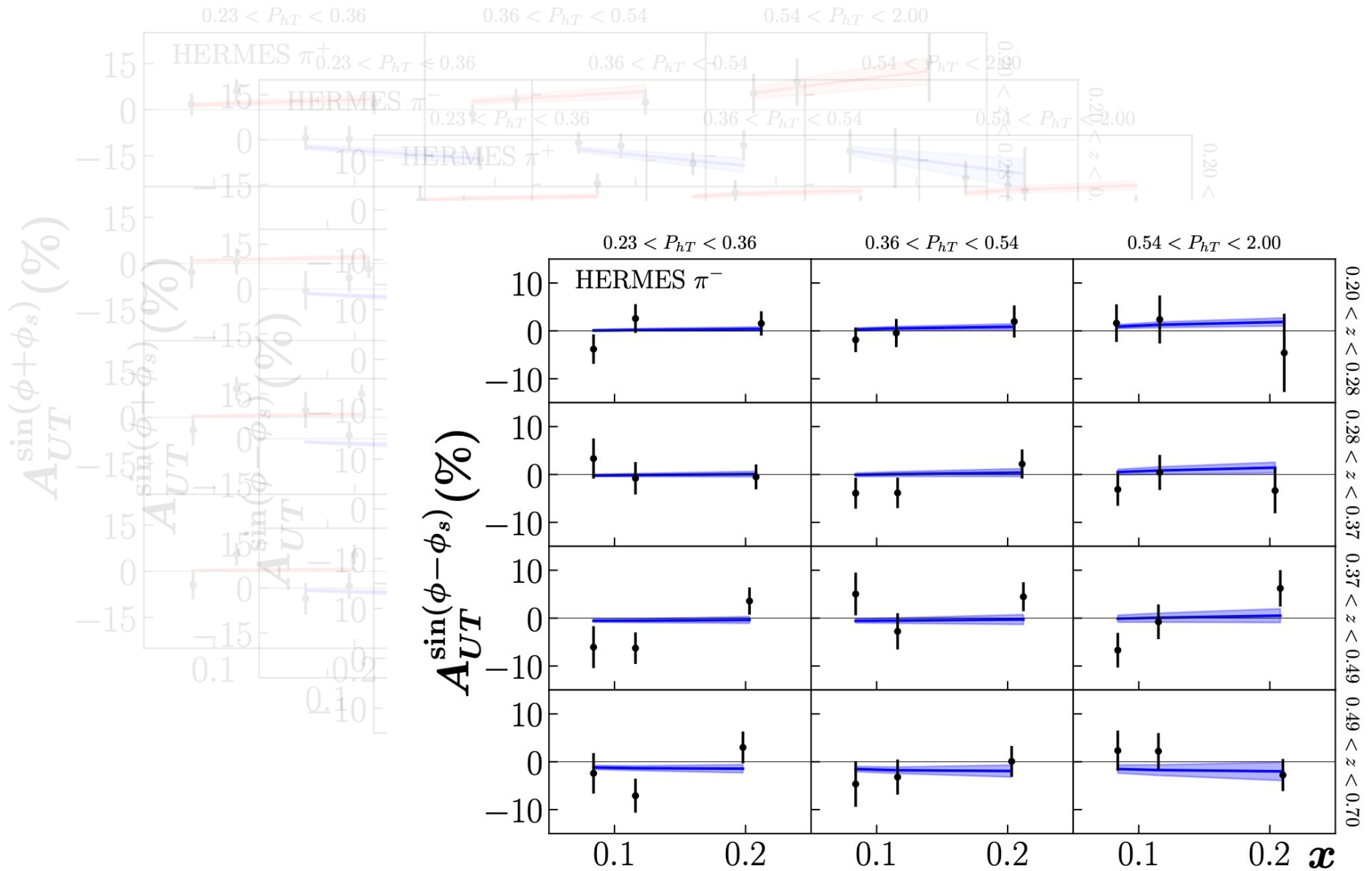
TABLE I: Summary of the observables analyzed in JAM3D-22 . There are a total of 21 different reactions. There are also a total of 8 non-perturbative functions when one takes into account flavor separation. The χ^2 is computed based on calculating for each point the theory expectation value from the replicas. *For the $A_{UT}^{\sin \phi_S}$ data we only use the x - and z -projections.

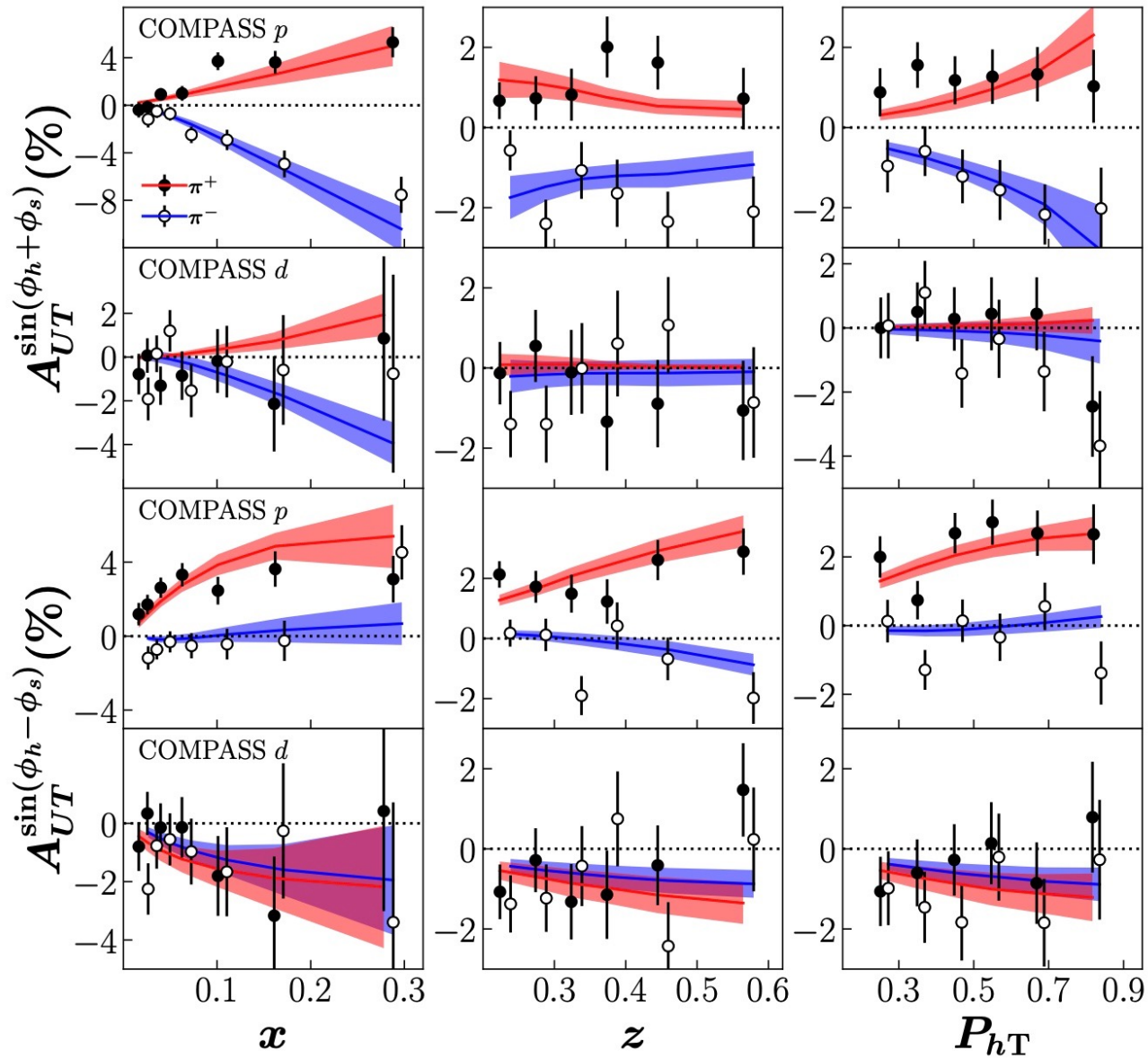


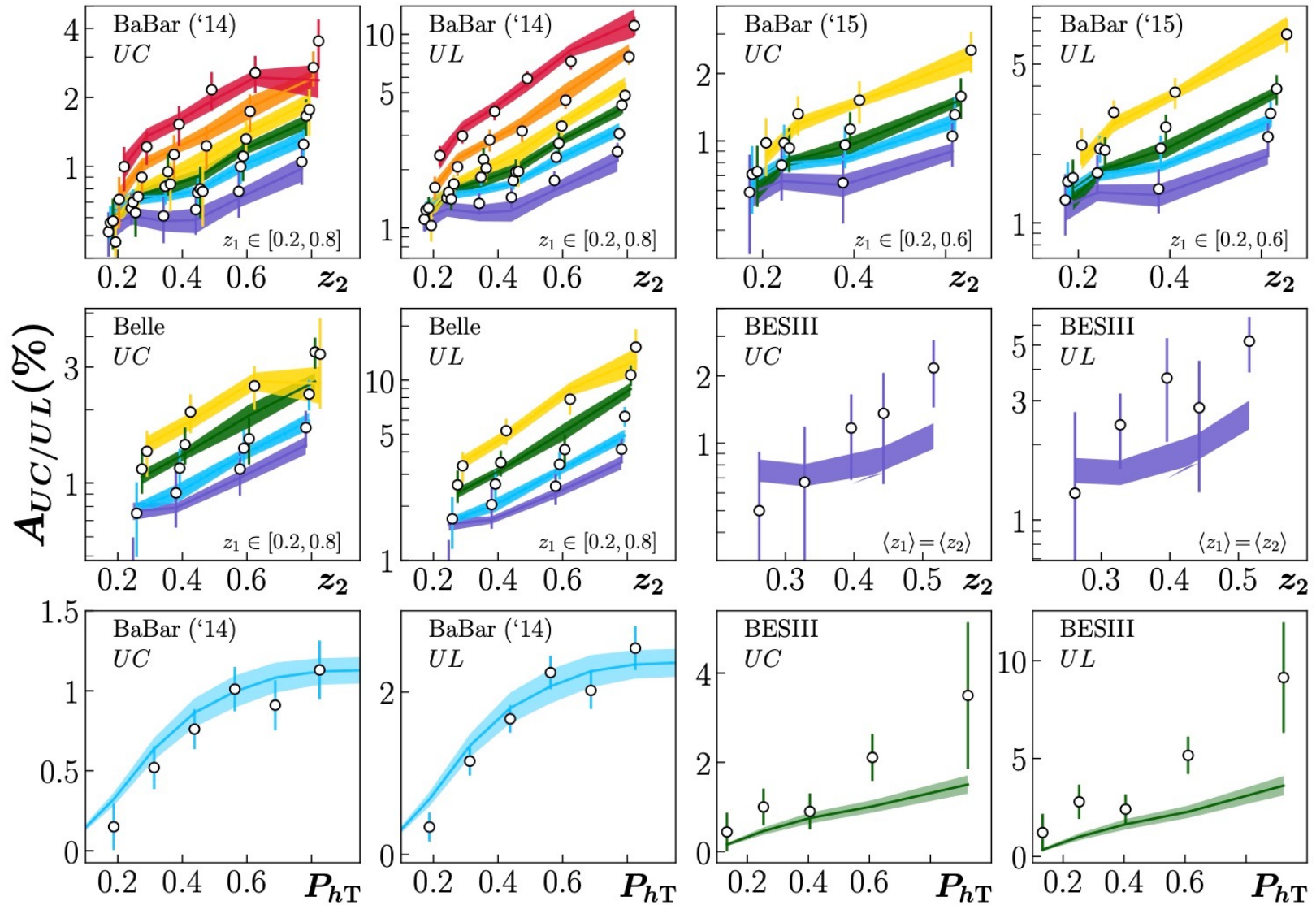


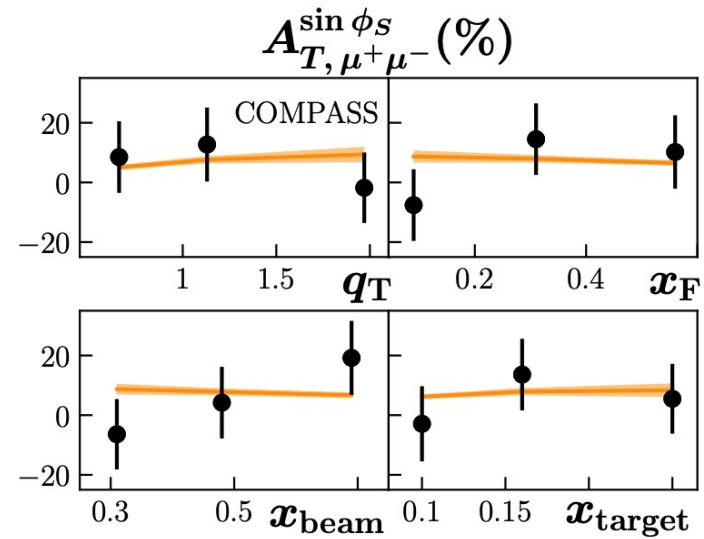
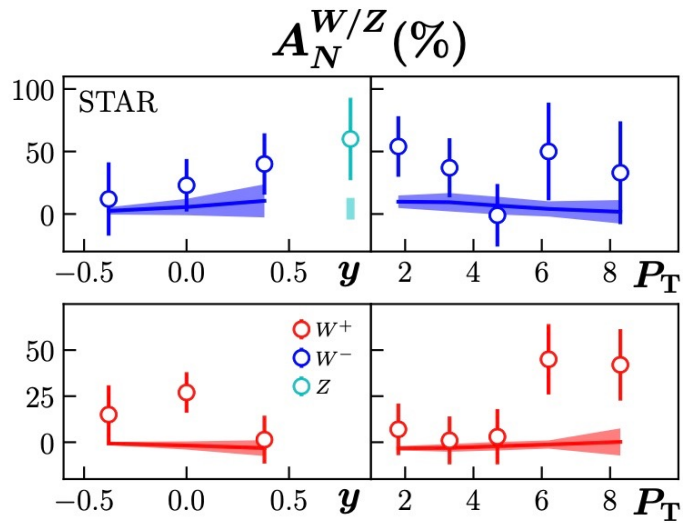






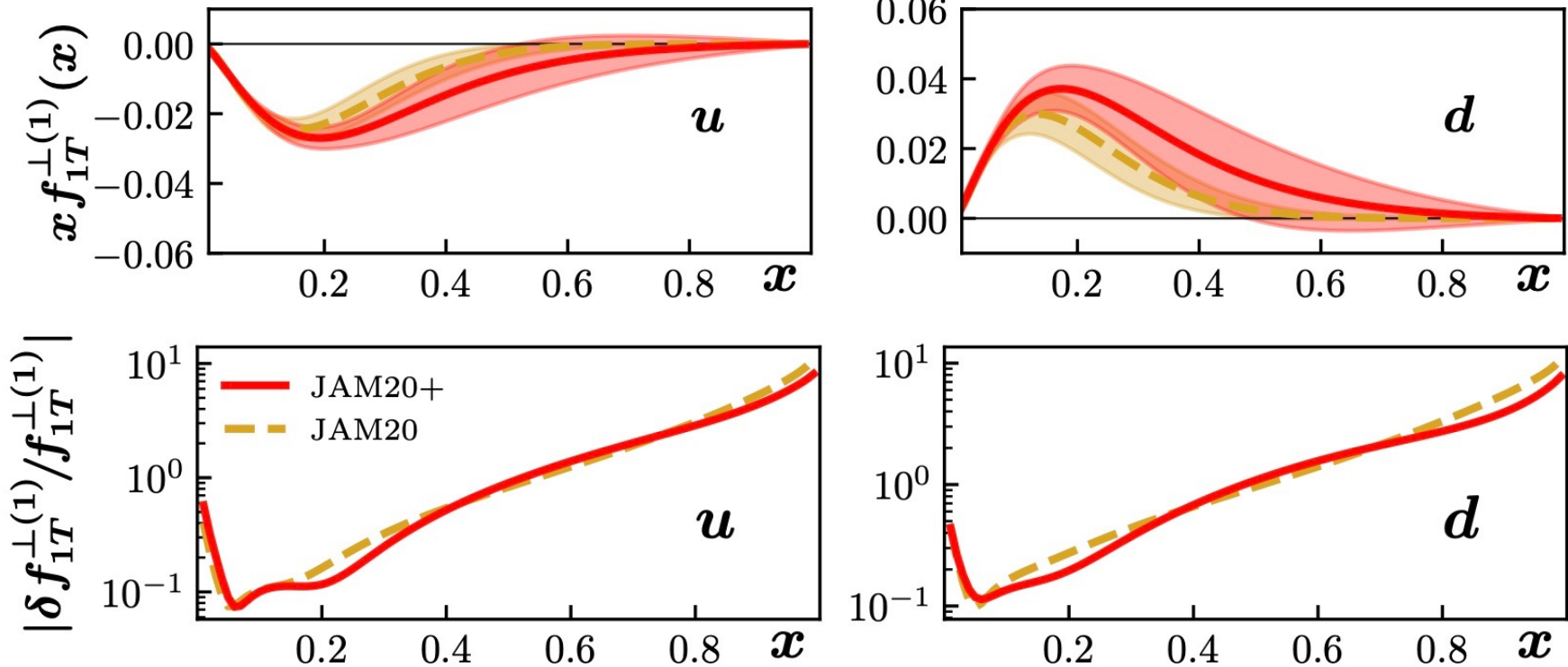






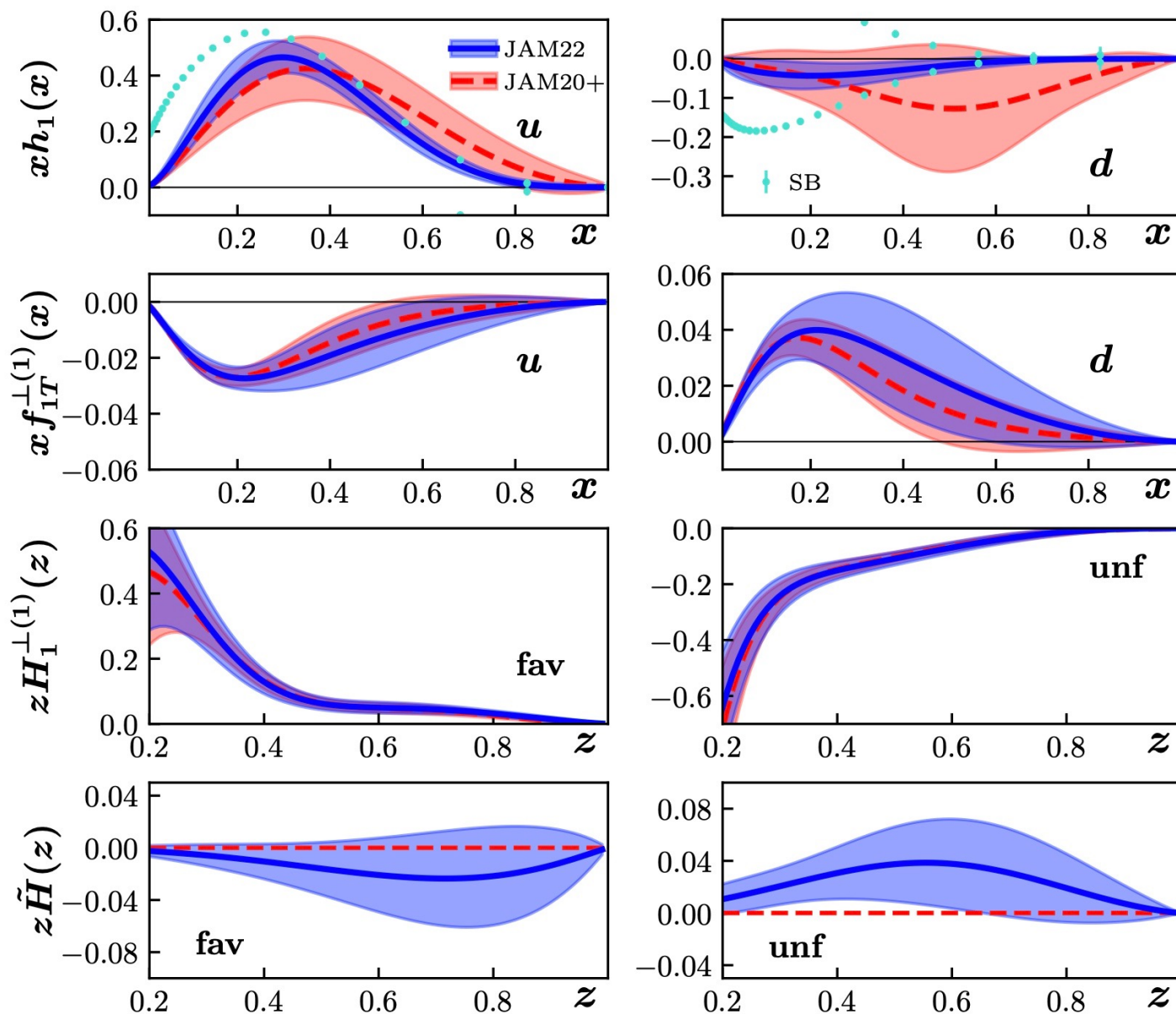
JAM3D-20+

Replace the Siverts effect and Collins effect HERMES data from JAM20 with their superseding 3D-binned version. No other constraints/data are added yet.



- Only the Siverts function changes size and shape but the relative uncertainty remains the same

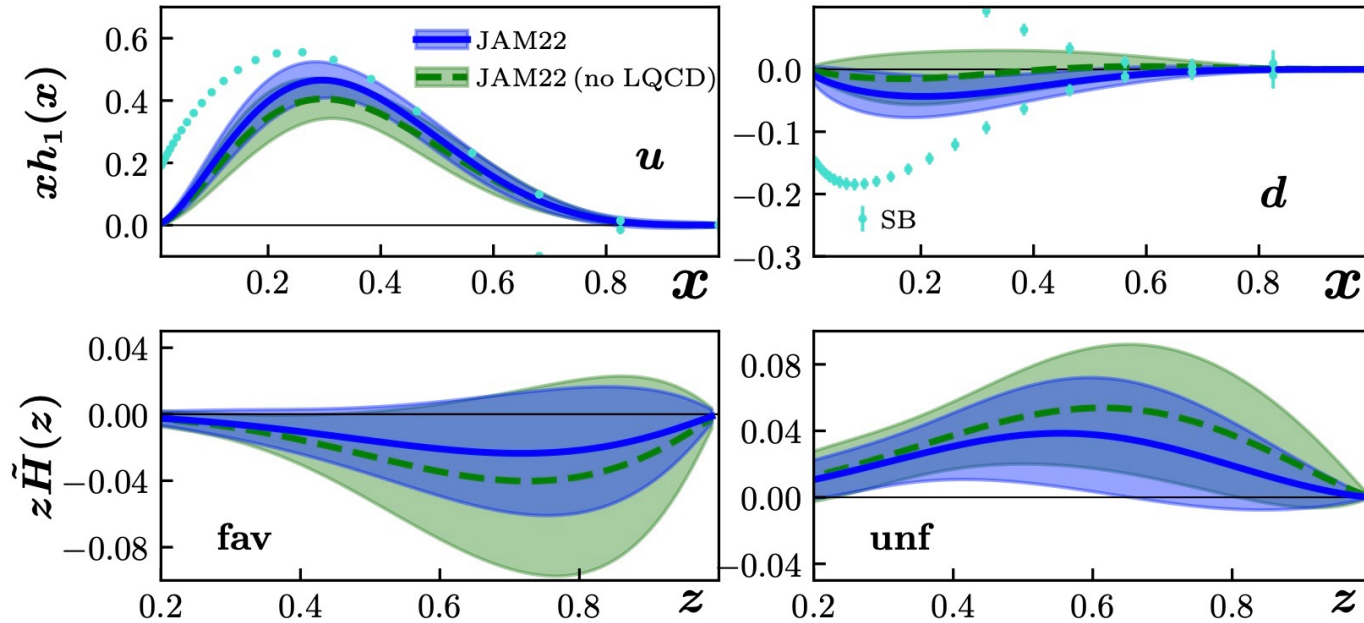
JAM3D-22



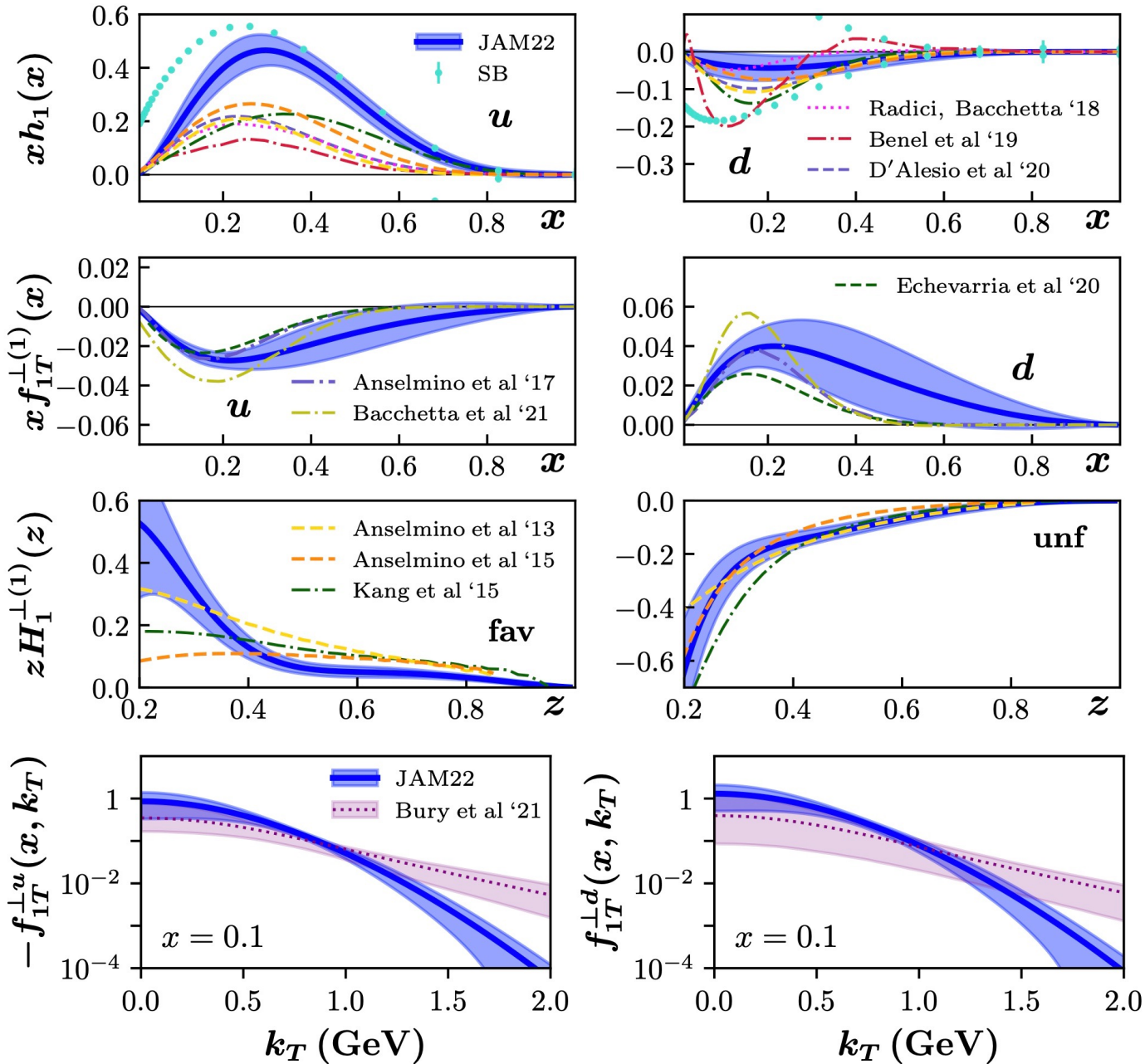
First direct information from experiment on $\tilde{H}(z)$

➤ Comments on the non-perturbative functions:

- Transversity becomes much more tightly constrained by now imposing the SB and including the lattice g_T data point, in particular the latter

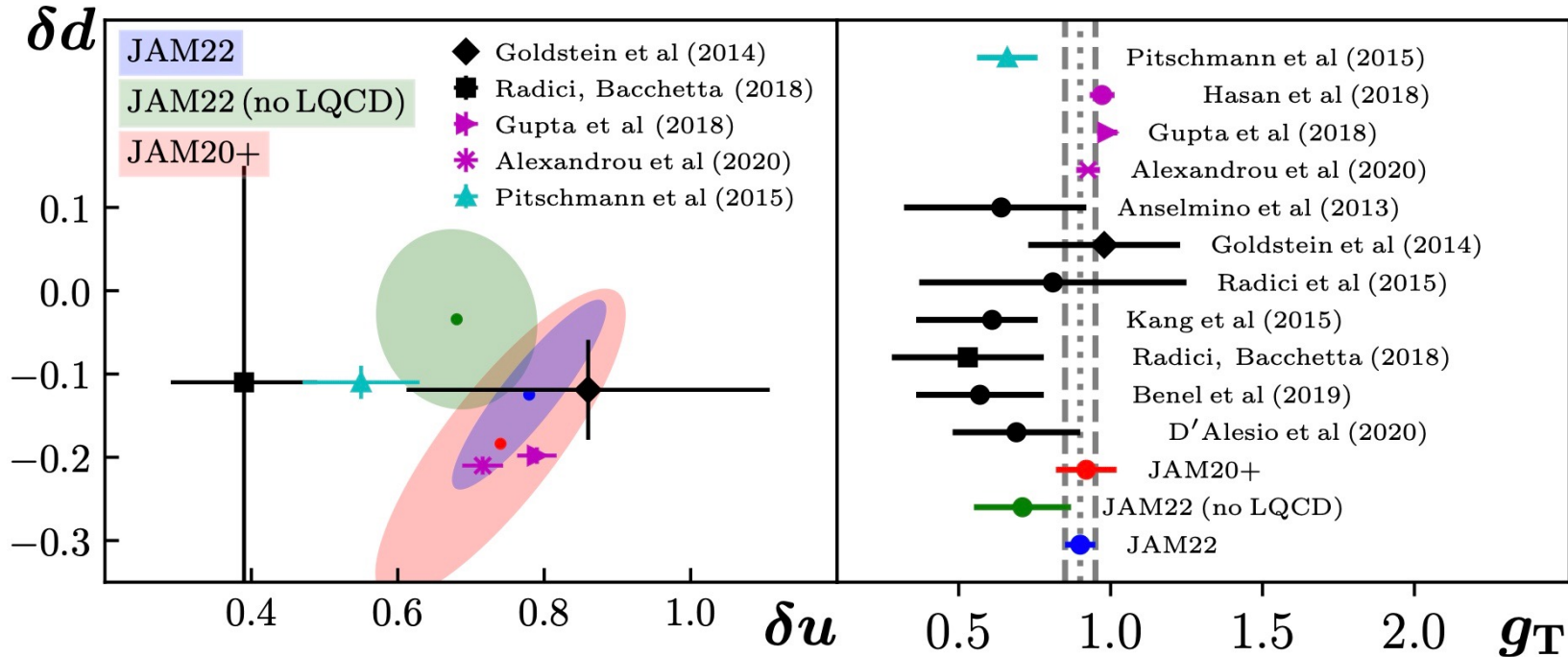


- Collins and Sivers functions remain basically the same from JAM3D-20+
- $\tilde{H}(z)$ behaves similar to the Collins function (favored and unfavored roughly equal in magnitude but opposite in sign) - expected since both are derived from the same underlying quark-gluon-quark FF (Kanazawa, et al. (2016))

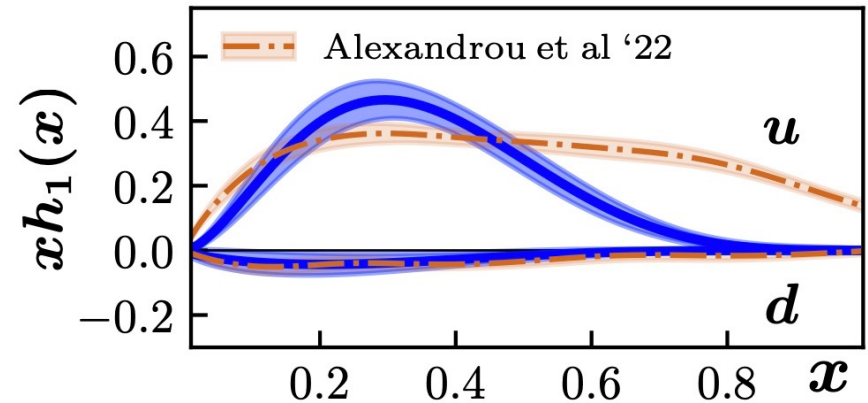
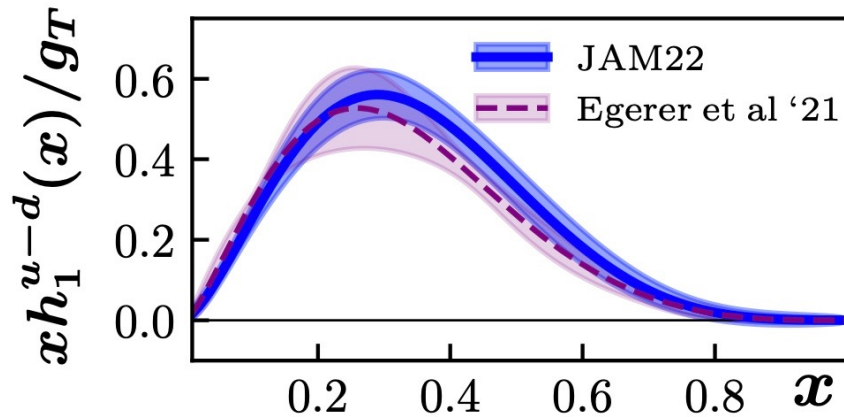


Disclaimer: only central curves of the 68% CL error band are shown

- Comments on comparison to other groups:
 - Now that we impose the Soffer bound, the down quark transversity is similar to other extractions
 - The *up quark transversity saturates the Soffer bound* due to the inclusion of proton-proton A_N data - analyses that only use TMD or dihadron data do *not* find this solution
 - The magnitude of the Sivers function first moment (function of x) is similar to recent extractions from Bacchetta, et al. and Echevarria, et al. but the fall off with x is slower in JAM3D-22 (due to using 3D-binned HERMES data)
 - Bury, et al. do *not* directly extract the Qiu-Sterman function but rather $\tilde{f}_{1T}^\perp(x, b_T)$
 - comparison with the F.T. agrees well at small k_T
 - at large k_T , effects from gluon radiation cause a deviation



- The tensor charge extractions are more precise from including the lattice g_T data point
- Note that because of the SB, one initially finds more tension with lattice, but this does *not* imply phenomenology and lattice are incompatible – one can only fully answer this by including lattice data in the analysis
- Once the the lattice g_T data point is included, we find the non-perturbative functions can accommodate it along with the experimental data



➤ Comments on comparison to other lattice calculations of transversity:

- The raw lattice data for Egerer, et al. and Alexandrou, et al. are compatible, but the former uses pseudo-PDFs and the latter quasi-PDFs
- The behavior at large x for the up quark in Alexandrou, et al. is due to systematics in the reconstruction of the x dependence in the quasi-PDF approach
- We find good agreement with lattice calculations of transversity
- *Now that the lattice g_T data point is included in JAM3D-22, the uncertainties in the phenomenological extraction of transversity are comparable with lattice*

Conclusions

- In 2020 we performed the first global analysis of SSAs in SIDIS, DY, e^+e^- annihilation, and proton-proton collisions and extracted a universal set of non-perturbative functions, showing a common origin of SSAs.
- We have updated our analysis using new data from HERMES (3D-binned Collins and Sivers effects and $A_{UT}^{\sin\phi_S}$) as well as constraints from lattice QCD (tensor charge g_T) and the Soffer bound on transversity.
- Our results show it is still possible to accommodate these data/constraints and describe all SSAs. The newly extracted transversity function and associated tensor charges are much more precise. We also have the first direct information from experiment on $\tilde{H}(z)$.
- Another update will include new STAR pion A_N data, jet A_N data (A_{NDY} , STAR) and hadron-in-jet Collins effect data (STAR).