This work is supported by the NSF under grant PHY 1653405 "CAREER: Constraining Parton Distribution Functions for New-Physics Searches"
Outline

§ Consumer’s Guide to Lattice Hadron Calculations

☞ Nucleon structure with controlled systematics in the physical limit \((m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty)\)

§ Bjorken-\(x\) Dependent Structure

☞ Nucleon Parton Distribution Functions (PDFs)
☞ Meson PDFs
☞ Meson Distribution Amplitude (DAs)

Apologies to those whose results I cannot cover due to time constraints
What is Lattice QCD?

Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories.

Physical observables are calculated from the path integral

\[
\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int DA \, D\bar{\psi} \, D\psi \, e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)
\]

in Euclidean space.

- Quark mass parameter (described by \( m_\pi \))
- Impose a UV cutoff
  - discretize spacetime
- Impose an infrared cutoff
  - finite volume

Recover physical limit

\[ m_\pi \to m_\pi^{\text{phys}}, \, a \to 0, \, L \to \infty \]
§ Lattice gauge theory was proposed in the 1970s by Wilson

.isOpen

Why haven’t we solved QCD yet?

§ Progress is limited by computational resources

1980s

Today

§ Greatly assisted by advances in algorithms

 isOpen

Physical pion-mass ensembles are not uncommon!
§ Pick a QCD vacuum

- Gauge/fermion actions, flavors (2, 2+1, 2+1+1), $m_\pi$, $a$, $L$, ...

Nucleon Matrix Elements

Huey-Wen Lin — 2020 JLUO Annual Meeting
§ Construct correlators (hadronic observables)

- Requires “quark propagator”
- Invert Dirac-operator matrix (rank $O(10^{12})$)

Lattice-QCD calculation of $\langle N | \bar{q} \Gamma q | N \rangle$
Lattice-QCD calculation of $\langle N| \bar{q} \Gamma q | N \rangle$

$t_i \quad t \quad O_i^q \quad t_f$

§ Careful analysis needed to remove systematics

★ Wrong results if excited-state systematic not controlled

$\tau : -5 \quad 0 \quad 5$

$t_f - t_i = 0.96 \text{ fm}$

$t_f - t_i = 1.2 \text{ fm}$

$t_f - t_i = 0.96 - 1.56 \text{ fm}$
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N | \bar{q} \Gamma q | N \rangle$

§ Systematic uncertainty (nonzero $a$, finite $L$, etc.)

- Excited-state contamination
- Extrapolation to the continuum limit
  $(m_{\pi} \rightarrow m_{\pi}^{\text{phys}}, L \rightarrow \infty, a \rightarrow 0)$
Precision Nucleon Couplings

§ $g_T$: zeroth moment of transversity

$\Gamma = \sigma_{\mu\nu}$

$g_T = \int_{-1}^{1} dx \, \delta q(x)$

§ A state-of-the-art calculation (PNDME)

♀ Extrapolate to the physical limit

First extrapolation to the physical limit of a nucleon matrix element!
Finally adopted by FLAG!


<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>$N_f$</th>
<th>Continuum extrapolation</th>
<th>Chiral extrapolation</th>
<th>Finite volume</th>
<th>Renormalization</th>
<th>Excited states</th>
<th>$g_T^{u-d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNDME 18</td>
<td>[84]</td>
<td>2+1+1</td>
<td>A</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>0.989(32)(10)</td>
</tr>
<tr>
<td>PNDME 16</td>
<td>[830]</td>
<td>2+1+1</td>
<td>A</td>
<td>O</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>0.987(51)(20)</td>
</tr>
<tr>
<td>PNDME 15</td>
<td>[828, 829]</td>
<td>2+1+1</td>
<td>A</td>
<td>O</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.020(76)</td>
</tr>
<tr>
<td>PNDME 13</td>
<td>[827]</td>
<td>2+1+1</td>
<td>A</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>1.047(61)</td>
</tr>
</tbody>
</table>

| Collaboration | Ref. | $N_f$ | | | | | | |
|---------------|------|-------| | | | | | |
| Mainz 18      | [915]| 2+1  | C                       | ★                   | ★             | ★               | ★              | 0.979(60)    |             |
| JLQCD 18      | [839]| 2+1  | A                       | O                   | ★             | ★               | ★              | 1.08(3)(3)(9)|             |
| LHPC 12       | [920]| 2+1  | A                       | O                   | ★             | ★               | ★              | 1.038(11)(12)|             |
| RBC/UKQCD 10D | [834]| 2+1  | A                       | ★                   | ★             | ★               | ★              | 0.9(2)       |             |

| Collaboration | Ref. | $N_f$ | | | | | | |
|---------------|------|-------| | | | | | |
| ETM 17        | [826]| 2    | A                       | O                   | O             | ★               | ★              | 1.004(21)(2)(19)|      |
| ETM 15D       | [822]| 2    | A                       | O                   | O             | ★               | ★              | 1.027(62)    |      |
| RQCD 14       | [819]| 2    | A                       | O                   | ★             | ★               | ★              | 1.005(17)(29)|      |
| RBC 08        | [918]| 2    | A                       | O                   | ★             | ★               | ★              | 0.93(6)      |      |

1 The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.
Improved transversity distribution with LQCD $g_T$

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^\pm$ production data from proton and deuteron targets

PDFs on the Lattice

§ Traditional lattice calculations rely on operator product expansion, only provide moments.

\[ \langle x^{n-1} \rangle_q = \int_{-1}^1 dx \ x^{n-1} q(x) \]

\[ \langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^1 dx \ x^{n-1} \Delta q(x) \]

\[ \langle x^{n-1} \rangle_{\delta q} = \int_{-1}^1 dx \ x^{n-1} \delta q(x) \]

most well known

very poorly known

§ True distribution can only be recovered with all moments.

spin-averaged/unpolarized

spin-dependent

longitudinally polarized

spin-dependent

transversely polarized
PDFs on the Lattice

§ Limited to the lowest few moments
☞ For higher moments, all ops mix with lower-dimension ops
☞ No practical proposal yet to overcome this problem

§ Relative error grows in higher moments
☞ Calculation would be costly
☞ Cannot separate valence contrib. from sea
PDFs on the Lattice

§ Limited to the lowest few moments
- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem
§ Relative error grows in higher moments
- Calculation would be costly
- Cannot separate valence contrib. from sea

New Strategy:
§ Adopt lightcone description for PDFs
§ Calculate finite-boost quark distribution
- In $P_z \to \infty$ limit, parton distribution recovered
- For finite $P_z$, corrections are applied through effective theory
§ Feasible with today’s resources!

Xiangdong Ji, PRL 111, 039103 (2013);
2004.03543

PDFs on the Lattice

§ Limited to the lowest few moments
- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem
§ Relative error grows in higher moments
- Calculation would be costly
- Cannot separate valence contrib. from sea

New Strategy:
§ Adopt lightcone description for PDFs
§ Calculate finite-boost quark distribution
- In $P_z \to \infty$ limit, parton distribution recovered
- For finite $P_z$, corrections are applied through effective theory
§ Feasible with today’s resources!

Xiangdong Ji, PRL 111, 039103 (2013);
2004.03543
Bjorken-\(x\) Dependent Structure

Nucleon PDFs

Quasi-PDF vs Pseudo-PDF
**Quasi-PDF vs Pseudo-PDF**

They both calculate the matrix element $h(z, P_z)$

§ **Quasi-PDF**

- No renormalization
- Renormalization and ratios $h^R(z, P_z, P^R)$ or $\frac{h(z,P_z,P^R)}{h(z=0,P_z,P^R)}$
- FT $zP_z$-space to $x$-space at fixed $z^2$
- FT $z$-space to $x$-space at fixed $P_z$

§ **Pseudo-PDF**

- No renormalization
- FT $zP_z$-space to $x$-space at fixed $z^2$

**$Q^2 = 0$**

Huey-Wen Lin — 2020 JLUO Annual Meeting
Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$

$p(P_z)$

$Q^2 = 0$

Γ

§ Pseudo-PDF

Pseudo-PDF

Plot by Yi-Bo Yang

§ Quasi-PDF

RI/MOM quasi-PDF

Slide from 2017
Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today

\[ \sum \] Wanted PDFs, GPDs, etc.

\times pQCD-calculated kernel

- Quasi-PDF/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, ... 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- ...

Huey-Wen Lin — 2020 JLUO Annual Meeting
Kernel is a complicated object; mostly only calculated up to one-loop level

Inverse problem to extract the wanted distribution
- Slightly different approaches from each group
- Systematics vary

Large momentum is needed in the lattice calculations in all methods to reach small-$x$ region
- Current projects focus on mid- to large-$x$
Quasi-PDF: two collaborations’ results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed

Not using parametrization (e.g. $xf(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.
Physical Pion Mass Results

§ Quasi-PDF: two collaborations’ results at physical pion mass

Boost momenta $P_z \leq 1.4$ GeV

Study of systematics still needed

Updated results at physical pion mass

Not using parametrization (e.g. $\mu_0 = a_0 x + a_1 x^2$

Less pretty results; less likely to exactly coincide with global fits.

$u(x) - d(x)$

---CT14

---matched PDF

1803.04393 (LP$^3$)

Updated results at physical pion mass
Physical Pion Mass Results

§ Summary of physical pion mass results

✧ Recent study increase boost momenta $P_z > 3$ GeV

$$u(x) - d(x)$$

<table>
<thead>
<tr>
<th>PDF Type</th>
<th>Study</th>
<th>$P_{\text{max}}$</th>
<th>2006.08636</th>
<th>LHCC Report 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>qPDF</td>
<td>LP3'18</td>
<td>$</td>
<td>P_{\text{max}}</td>
<td>= 3.0$ GeV</td>
</tr>
<tr>
<td>pPDF</td>
<td>ETMC'20</td>
<td>$</td>
<td>P_{\text{max}}</td>
<td>= 1.4$ GeV</td>
</tr>
<tr>
<td>qPDF</td>
<td>ETMC'18</td>
<td>$</td>
<td>P_{\text{max}}</td>
<td>= 1.4$ GeV</td>
</tr>
<tr>
<td>pPDF</td>
<td>JLab/W&amp;M'20</td>
<td>$</td>
<td>P_{\text{max}}</td>
<td>= 3.3$ GeV</td>
</tr>
</tbody>
</table>

$u_v(x) - d_v(x)$

2006.08636, PDFLattice2019 report
§ Summary of physical pion mass results

Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$

Finite volume, Discretization, ...

2006.08636, PDFLattice2019 report
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

\[ \bar{d}(x) - \bar{u}(x) \]

- qPDF: LP3'18, $|P_{\text{max}}| = 3.0$ GeV
- pPDF: ETMC'20, $|P_{\text{max}}| = 1.4$ GeV
- qPDF: ETMC'18, $|P_{\text{max}}| = 1.4$ GeV

2006.08636, PDFLattice2019 report
Summary of physical pion mass results

Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$

- qPDF: LP3'18, $|P_{max}| = 3.0$ GeV
- pPDF: ETMC'20, $|P_{max}| = 1.4$ GeV
- qPDF: ETMC'18, $|P_{max}| = 1.4$ GeV

2006.08636, PDFLattice2019 report
\section*{Summary of physical pion mass results}

\textit{Quasi-PDF} method only

\[ \Delta u(x) - \Delta d(x) \]

Finite volume, Discretization, ...

\[ \Delta \bar{u}(x) - \Delta \bar{d}(x) \]

2006.08636, PDFLattice2019 report
§ Summary of physical pion mass results

Quasi-PDF method only

$$\Delta u(x) - \Delta d(x)$$

Helicity long. polarized

$$\Delta \bar{u}(x) - \Delta \bar{d}(x)$$

Finite volume, Discretization, ...

2006.08636, PDFLattice2019 report
**Polarized PDFs**

§ Summary of physical pion mass results

✨ Quasi-PDF method only

**δu(x) − δd(x)**

Transversity

2006.08636, PDFLattice2019 report
Polarized PDFs

§ Summary of physical pion mass results

Quasi-PDF method only

\[ \delta u(x) - \delta d(x) \]

\[ \delta \bar{d}(x) - \delta \bar{u}(x) \]

Finite volume, Discretization, ...

2006.08636, PDFLattice2019 report
Gluon PDF in Nucleon

Pioneering first glimpse into gluon PDF using LaMET

- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved


(plot by Zhouyou Fan)
§ Large uncertainties in global PDFs

Assumptions imposed due to lack of precision data

\[ s = \bar{s} = \kappa (\bar{u} + \bar{d}) \]

CTEQ-JLAB vhttps://www.jlab.org/theory/cj/

§ Results by MSULat/quasi-PDF method 2005.12015, Zhang, Lin, Yoon

 Clover on 2+1+1 HISQ, 
 \[ \alpha \approx 0.12 \text{ fm} \]
 extrapolated to \( M_\pi \approx 140 \text{ MeV} \)

(plot by Rui Zhang)
§ First finite-volume study in quasi-PDFs

 Clover on 2+1+1 HISQ, $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

 $M_\pi L \approx 3.3, 4.4, 5.5, P_z \approx 1.3$ GeV

Systematics Study

§ Finite-volume study in unpolarized pseudo-PDFs

$\bowtie$ 2+1f clover, $M_\pi \approx 415$ MeV, $a \approx 0.127$ fm

$\bowtie$ Two volumes used: $L \approx 3, 4.5$ fm

B. Joo et al (Jlab/W&M) 1908.09771

§ Also see strong lattice-spacing dependence

§ Lattice artifacts are sensitive to the simulated QCD vacuum

$\bowtie$ Each group will have to check their own systematics carefully
Superfine Lattice Spacing

§ Approaching continuum limit in quasi-PDFs

❖ Important for all $x$-dependent methods
Large momentum required to reach $x < 0.1$ reliably
$(aP_z)^n$ systematics should be small
❖ First work done with superfine lattice spacing, $a \approx 0.042$ fm

Unpolarized ME 2005.12015, MSU/BNL Polarized ME

(plot by Xiang Goa)
Bjorken-x Dependent Structure

Meson Structure

\[ K(P_z) \]

\[ Q^2 = 0 \]

\[ t_i \quad t_{\text{sep}} \quad t_f \]
§ Status as of Summer 2019

\[ M_\pi \approx 310 \text{ MeV} \]

\[ M_\pi \approx 426 \text{ MeV} \]

LSC

§ Single-ensemble calculation

\[ f^{\pi}(x) \approx 1.1 = 0 \]

Non-physical pion mass, single lattice spacing, single volume
Pion Valence-Quark PDF

§ Results from JLab-W&M/LSC method

\( M_\pi = 278, 358, 413 \text{ MeV with } a = 0.094, 0.127 \text{ fm} \)

\( \text{Extrapolated to physical limit (shown as blue band)} \)

\( \text{Renormalized } Z_{V,A} \text{ in RI/MOM, matched to } \overline{\text{MS}}, \text{ run to } 27 \text{ GeV}^2 \)


\( q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1, \beta+1) + \gamma B(\alpha+2, \beta+1)} \)

\( \beta = 1.24(22)_{\text{stat}}(7)_{\text{sys}} \)

\( \beta = 2.12(56)_{\text{stat}}(14)_{\text{sys}} \)

\( \mu^2 = 27 \text{ GeV}^2 \)
§ Results from MSULAT/quasi-PDF method

- $M_\pi = 220, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- Extrapolated to physical limit (shown as pink/green band)
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV

H. Lin et al., 2003.14128

- C. Chen et al, PRD93, 074021 (2016), 1602.01502.

First Kaon PDFs

§ Kaon-to-pion ratio for up-quark PDF

§ Kaon strange-quark PDF

_extrapolated to physical limit_

H. Lin et al., 2003.14128

J. Lan, et al., PRL122, 172001 (2019), 1901.11430;
T. Nguyen et al., PRC83, 062201 (2011), 1102.2448

A. Watanabe et al, PRD97, 074015 (2018), 1710.09529.
Pion Gluon PDF

§ Pioneering first glimpse into pion gluon PDF using LaMET

▶ Promising results using coordinate-space comparison, but signal does not go far in $z$


▶ Lattice calculation #1:
 overlap/2+1DWF, 0.16 fm, 340-MeV sea pion mass

▶ Lattice calculation #2:
clover/2+1+1 HISQ, 0.15 fm, 310-MeV sea pion mass with increased momenta (normalized by $\langle x \rangle_g$)

$M_\pi = 678$ MeV

$M_\pi = 685$ MeV

(plot by Zhouyou Fan) $zP_z$
Meson DAs in Quasi-PDFs

Rui Zhang (MSU)  
Carson Honkala (MSU)  
Jiunn-Wei Chen (NTU)  
+ HWL

R. Zhang et al. (MSULat), 2005.13955
The first continuum-limit study of $x$-dependent meson DA on the lattice

$\mathbf{Pion and Kaon DA}$

\[ M_\pi \in \{310, 690 (\eta_s)\} \text{ MeV} \]
\[ a \in \{0.06, 0.09, 0.12\} \text{ fm} \]
\[ M_{\pi}^{\text{min}} L = 4.5 \]

\[
C_M^{\text{DA}}(z, P, t) = \left\langle 0 \left| \int d^3y \, e^{i \vec{P} \cdot \vec{y}} \bar{\psi}_1(\vec{y}, t) \gamma_z \gamma_5 U(\vec{y}, \vec{y} + z \hat{z}) \psi_2(\vec{y} + z \hat{z}, t) \bar{\psi}_2(0,0) \gamma_5 \psi_1(0,0) \right| 0 \right\rangle
\]
§ Extract the DA distribution from the physical-continuum matrix elements

\[ h(z, \mu^R, p^R_Z, P_z) = \int_{-\infty}^{\infty} dx \ e^{i(1-x)zP_z} \int_0^1 dy \ C\left( x, y, \left( \frac{\mu^R}{P_z^R} \right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{P^R_Z} \right) f_{m,n}(y) \]

\[ f_{m,n}(x) = \frac{1}{B(m+1, n+1)} x^m (1-x)^n \]

\[ B(m+1, n+1) = \int_0^1 dx \ x^m (1-x)^n \]

**1st method: fit to the functional form**

---

**Pion**

---

**Kaon**
Machine Learning - A Promising Solution?

Machine learning models are effective in extracting complicated dependence of the output data on input data.

\[
h(z, \mu \Omega, p \Omega, \bar{P} \Omega) = \int_{-\infty}^{\infty} e^{-ixz} \int_{-\infty}^{1} e^{-yCx, y} \int_{-\infty}^{1} \int_{-\infty}^{1} f_{m, n} x^m y^n \mu_{\Omega} P_{\Omega} \mu_{\Omega} P_{\Omega} = B_{m+1, n+1} + B_{m+1, n+1} x^m y^n
\]

Slide by Rui Zhang
Pion and Kaon DA

Machine Learning - preliminary attempts

Training: 10,000 Pseudo-data from functional form $x^a(1 - x)^b/B[a + 1, b + 1]$ with $a, b > 0$. With random relative noise added.

Extrapolation test: Generate $h(z)$ from $f(x) = N \sin^a(\pi x)$ with $a = 0.5, 1, 2$. Add 1000 random noise $\sigma(z) = 0.1 e^{0.1z} h(z)$ to $h(z)$. Estimate mean and error of the prediction on 1000 samples.
§ Extract the DA distribution from the physical-continuum matrix elements

\[ h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_{0}^{1} dy \ C(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}) f_{m,n}(y)e^{i(1-x)zP_z} \]

\( 2^{\text{nd}} \) method: use machine learning to determine \( f \)
§ Extract the DA distribution from the physical-continuum matrix elements

\[ h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_{0}^{1} dy \ C(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R})f_{m,n}(y)e^{i(1-x)zP_z} \]
§ Extract the DA distribution from the physical-continuum matrix elements

\[ h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_{0}^{1} dy \ C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{mn}(y) e^{i(1-x)zP_z} \]
Summary & Outlook

Exciting time for studying meson structure on the lattice

§ Overcoming longstanding obstacle to full x-distribution

☞ Most importantly, this can be done with today’s computers

§ Nucleon PDF results from both quasi- and pseudo-PDF

§ Progress made in pion and kaon structure

☞ First look at kaon PDF, and pion GPD

☞ Continuum-limit pion and kaon DAs

§ Future improvement

☞ Lattice systematics, reduce the parameter-dependence of distribution functions, etc.

Thanks to MILC collaboration for sharing lattices and NSF CAREER Award under grant PHY 1653405
Backup Slides
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

행정 details: clover/HISQ, 0.12fm, 310-MeV pion mass

\[ P_z \approx 1.3, 1.6 \text{ GeV} \]

J. Chen, HL, J. Zhang, 1904.12376

\[ H_q^{\pi}(x, \xi, t, \mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^-P^+} \left( \pi(P + \Delta/2) \left| \bar{q} \left( \frac{\eta^-}{2} \right) \gamma^+ \Gamma \left( \frac{\eta^-}{2}, -\frac{\eta^-}{2} \right) q \left( -\frac{\eta^-}{2} \right) \right| \pi(P - \Delta/2) \right) \]

\[ P_z \approx 1.6 \text{ GeV} \]

\[ \xi = 0 \]

(plot by J. Zhang)
Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using LaMET

-Lattice details: twisted-mass fermions, 0.09fm, 270-MeV pion mass, \( P_z \approx 0.83 \) GeV

\[
F(x, \xi, t) = \int \frac{d\zeta^-}{4\pi} e^{-ixP^+\zeta^-} \langle P'|O_{\gamma^+}(\zeta^-)|P \rangle = \frac{1}{2P^+} \bar{u}(P') \left\{ H(x, \xi, t) + E(x, \xi, t) \frac{i\sigma^+ \Delta \mu}{2M} \right\} u(P)
\]

nucleon \( \xi = 0 \) isovector results

C. Alexandrou, (ETMC), 1910.13229 (Lattice 2019 Proceeding)
Select Lattice Structure Results

Future Prospects and Challenges Ahead
§ Impacts on antiquark and small-x regions?

☞ One needs large momentum just to get the sign of the antiquark correct!
☞ With small $zP_z$, one will miss over the majority of $x$.

Not just a quasi-PDF problem

☞ Going for large $P_z$ is an unavoidable direction for all $x$-dependent methods
☞ Higher-loop matching kernel is not going to do much for it!

$z_{max} = 0.72 \text{ fm}$

$P_z = 1.6 \text{ GeV}$
$P_z = 2.4 \text{ GeV}$
$P_z = 3.2 \text{ GeV}$
$P_z = 4.0 \text{ GeV}$
§ What lessons learned here?

▷ Given $L_z \approx 15$, you need large momentum to just get the sign of the antiquark correct!

▷ With small $zP_z$, you will miss over the majority of $x$

▷ Not just a quasi-PDF problem

▷ Going for large $P_z$ is an unavoidable direction for any method that requires steps similar to Fourier transformation.

$z_{\text{max}} = 2.88 \text{ fm}$

$P_z = 0.4 \text{ GeV}$

$P_z = 0.8 \text{ GeV}$

$P_z = 1.2 \text{ GeV}$

$P_z = 2.4 \text{ GeV}$

$z_{\text{max}} = 0.72 \text{ fm}$

$P_z = 1.6 \text{ GeV}$

$P_z = 2.4 \text{ GeV}$

$P_z = 3.2 \text{ GeV}$

$P_z = 4.0 \text{ GeV}$
§ What lessons learned here?

Given \( L_z \approx 15 \), you need large momentum to just get the sign of the antiquark correct!

With small \( z P_z \) you will miss over the majority of \( x \)

Not just a quasi-PDF problem

Going for large \( P_z \) is an unavoidable direction for any method that requires steps similar to Fourier transformation

Continuum Toy Models

Huey-Wen Lin — 2020 JLUO Annual Meeting

The \( x \)-dependent PDFs will be doomed by a bad choice of \( \text{max } z P_z \)!

(unless modeling \( z P_z \) dependence)

Higher-loop matching in LaMET later is not going to do much for it!

Reaching \( x < 0.1 \) for (anti)quark remains challenging especially without replying on an assumed parametrization transformation
Input
\[ X_i = (O_i^1, O_i^2, \ldots) \]

ML Model

Output
\[ \hat{O}_i \]


All Data

Labeled (lb) Data

Unlabeled (ul) Data

Bias-Correction (BC) Data

Training (tr) Data

Prediction with bias correction Yoon et al., PRD 2018:
\[
\langle C_{\text{pred},\text{BC}} \rangle = \langle C_{\text{pred}} \rangle_{ul} + \langle C_{\text{BC} - C_{\text{pred}}} \rangle_{BC}
\]
Machine-Learning Prediction

Input

\[ X_i = (O_i^1, O_i^2, \ldots) \]

ML Model

Output

\[ \hat{O}_i \]


§ Multiple quasi-PDF data sets studied (meson DA, gluon/kaon PDFs)

Example kaon PDF at 220-MeV ensemble

(plot by Rui Zhang)