UNIVERSITY OF CAMBRIDGE

The scattering of spinning hadrons from lattice QCD

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JLUO Annual Users Group meeting 2020

hadspec

hadspec.org

'natural' spin-parities: $J^P = 0^+, 1^-, 2^+, \ldots$

$f_0(500), \kappa(700), a_0(980)$

$\rho(770), K^*(892)$

$f_2(1270), a_2(1320), K_2^*(1430)$

'unnatural' spin-parities: $J^P = 0^-, 1^+, 2^-, \ldots$

$\eta'(958), \pi(1300)$

$h_1(1170), b_1(1235), a_1(1260)$

$\eta_2(1645), \pi_2(1670)$

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seen in pseudoscalar-pseudoscalar scattering

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forbidden in pseudoscalar-pseudoscalar scattering

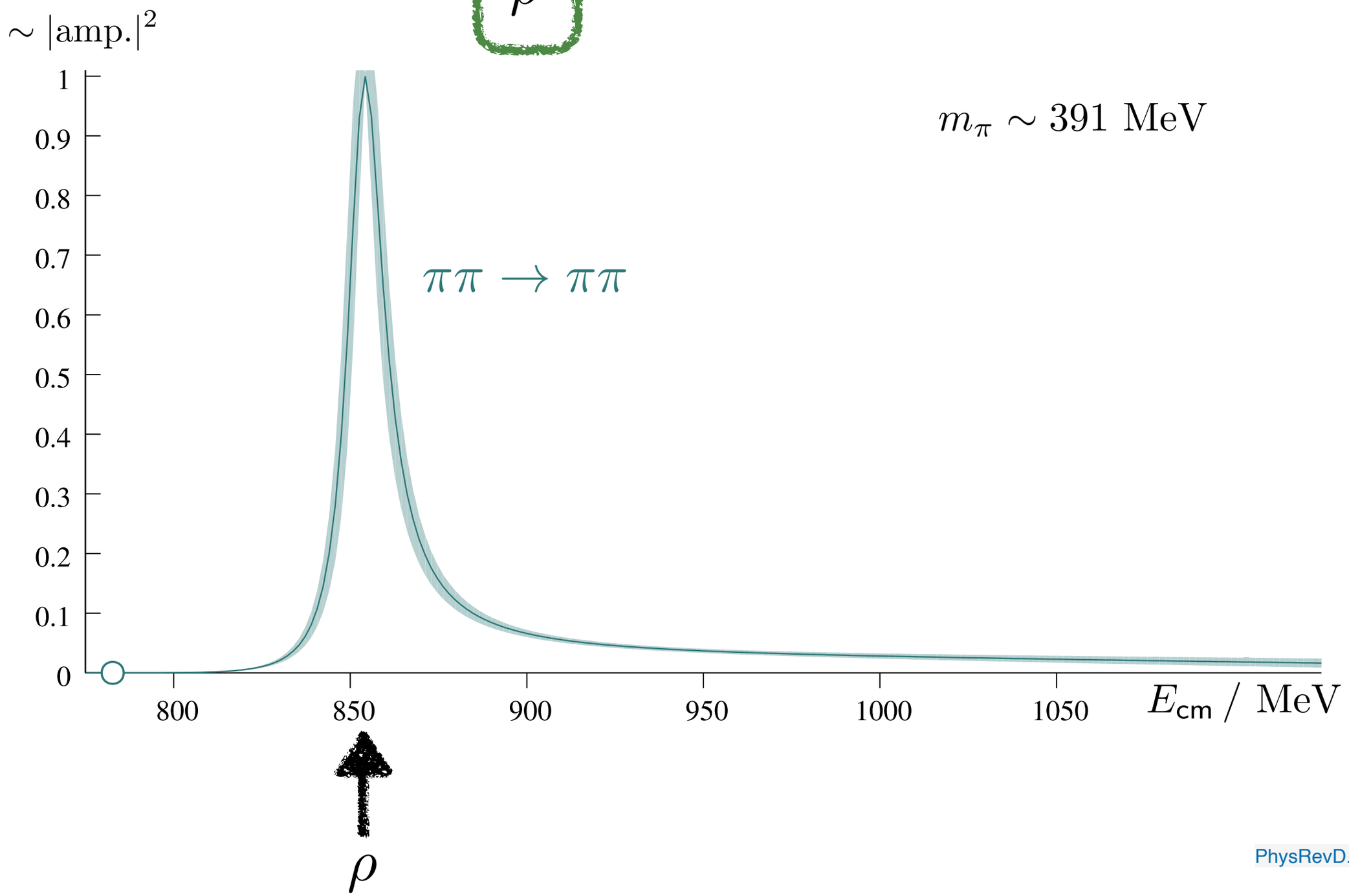
need something else, e.g. vector-pseudoscalar scattering…

LQCD very successful last decade in pseudoscalar-pseudoscalar scattering calculations:

Elastic scattering:

$$\pi\pi \to \pi\pi \qquad \boxed{\begin{matrix} {}^1P_1 \\ \rho \end{matrix}} \qquad J^P = 1^-$$

notation: ${}^{2S+1}\ell_J$



$m_\pi \sim 391$ MeV

PhysRevD.87.034505

Coupled-channel scattering:

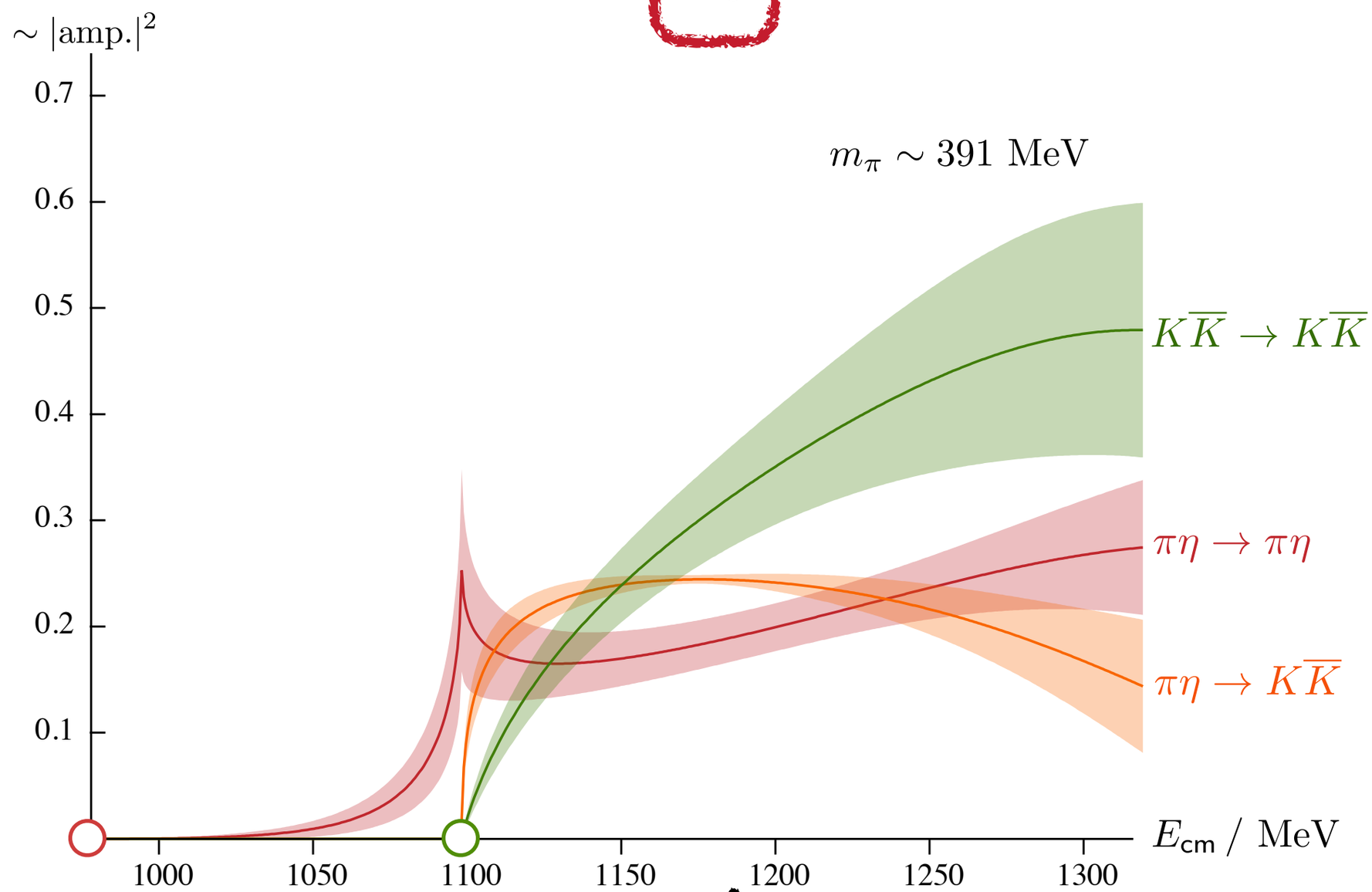
$$\{\pi\eta, K\overline{K}\} \to \{\pi\eta, K\overline{K}\}$$

$^1S_0$

a_0

notation: $^{2S+1}\ell_J$

$$J^P = 0^+$$



a_0

Many more examples in the charm sector...

PhysRevD.93.094506

Scattering hadrons with non-zero intrinsic spin much more complicated:

- Many partial-waves feature, e.g. vector-pseudoscalar scattering

$$1^- \otimes 0^- = {}^3S_1, \{ {}^3P_0 , {}^3P_1 , {}^3P_2 \} , \{ {}^3D_1, {}^3D_2 , {}^3D_3 \} , \cdots$$

`dynamically-coupled'

$$J^P = 1^+$$

- Resonances at higher energies, more coupled hadron-hadron channels

these are some of the challenges that I tackled in my thesis...

$b_1(1235)$

$I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV
Full width $\Gamma = 142 \pm 9$ MeV

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)
$\omega\pi$	seen [D/S amplitude ratio $= 0.277 \pm 0.027$]
$\pi^\pm\gamma$	$(1.6 \pm 0.4) \times 10^{-3}$
$\eta\rho$	seen
$\pi^+\pi^+\pi^-\pi^0$	< 50 %
$K^*(892)^\pm K^\mp$	seen
$(KK)^\pm\pi^0$	< 8 %
$K^0_S K^0_L \pi^\pm$	< 6 %
$K^0_S K^0_S \pi^\pm$	< 2 %
$\phi\pi$	< 1.5 %

PDG

very interesting resonance and great for a lattice calculation…

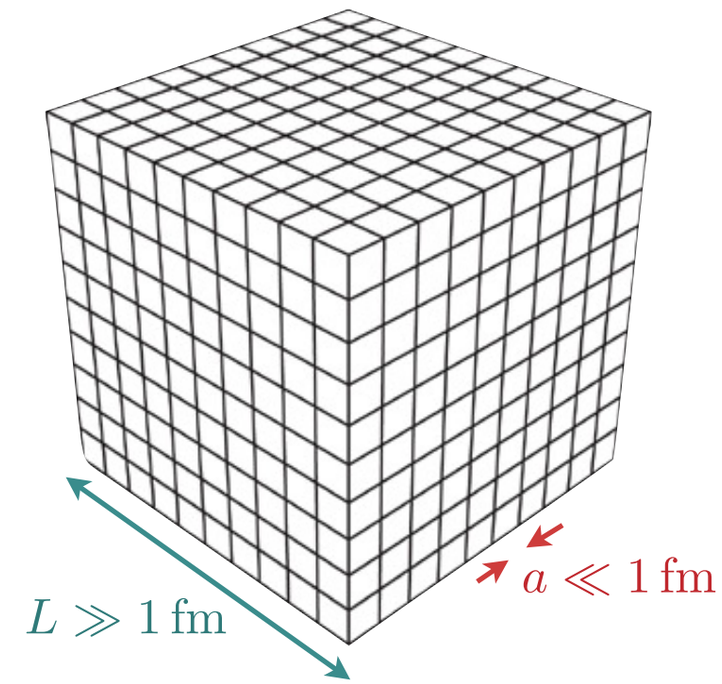
what is lattice QCD?

$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \, f(\psi,\bar{\psi},A_\mu)\, e^{-S^{\mathrm{Euc}}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

sum over quark/gluon field configurations

operators of interest

probability for a field configuration



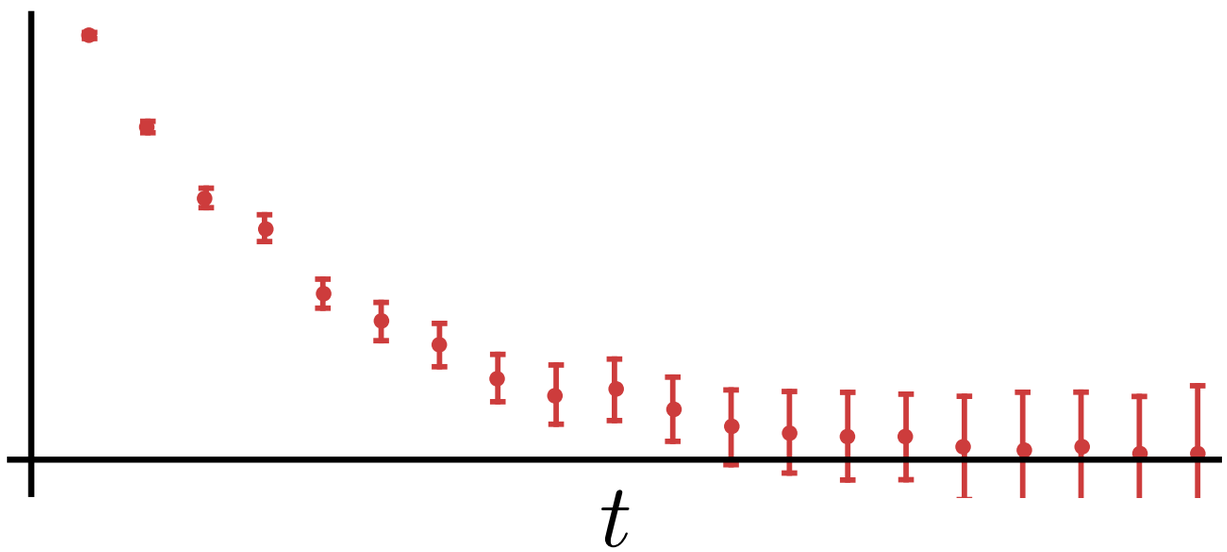
lattice QCD is QCD under controlled approximations:

- finite lattice spacing
- finite spacetime volume (a finite box)
- choice of action parameters, including quark mass

we use lattice QCD as a means to compute correlation functions…

$$\langle 0 | \mathcal{O}_f(t)\, \mathcal{O}_i^\dagger(0) | 0 \rangle$$

e.g. $\mathcal{O}(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\, \bar{\psi}_{\vec{x}t}\gamma_5\psi_{\vec{x}t}$



finite-volume spectra

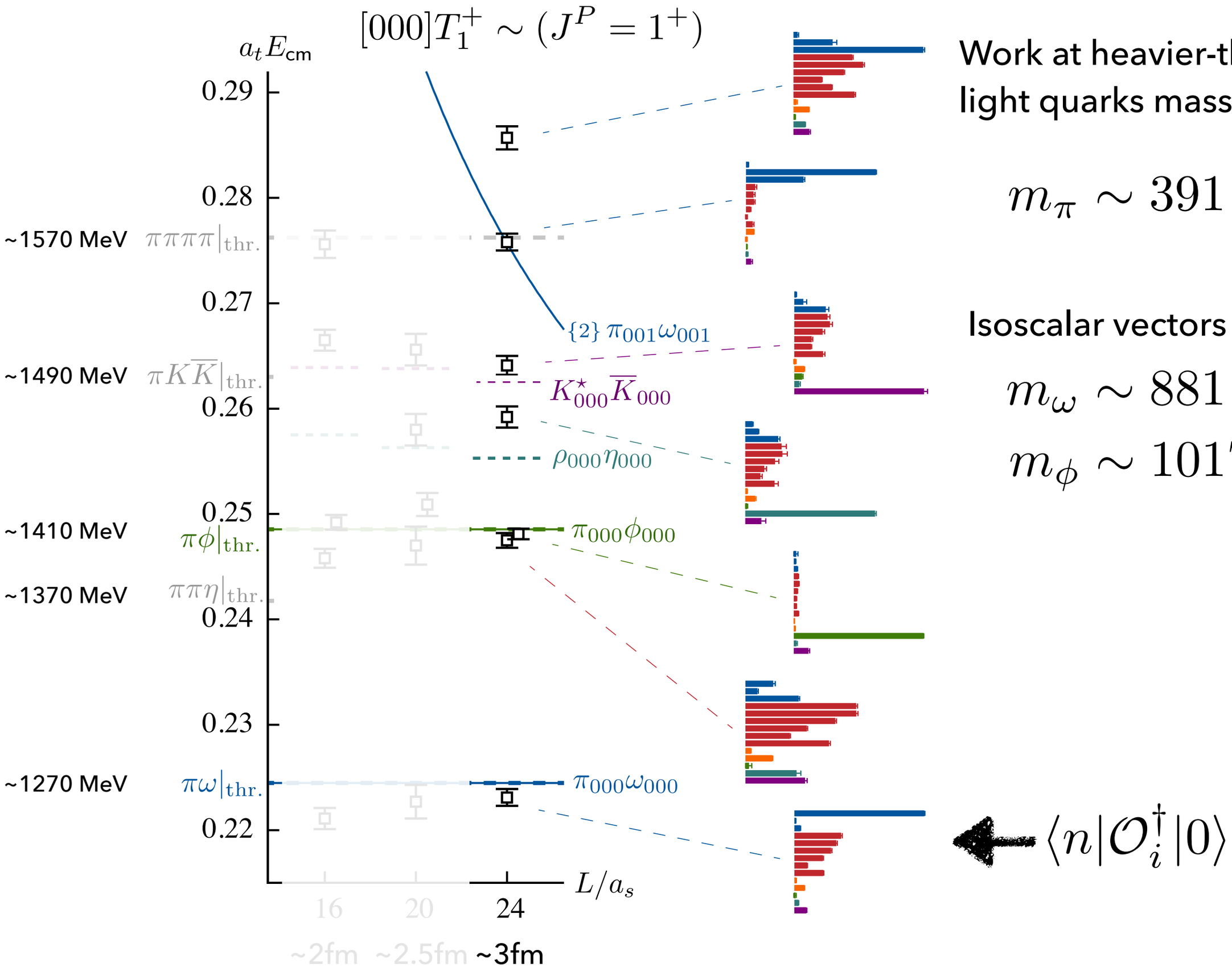
$$= \sum_{\mathfrak{n}} \langle 0 | \mathcal{O}_f | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_i^\dagger | 0 \rangle e^{-E_{\mathfrak{n}} t}$$

operator-state overlap

`how well does the operator interpolate state $\mathfrak{n}$'

variational method:

- compute a large matrix of two-point correlation functions with desired quantum numbers
- diagonalise the matrix and obtain the excited finite-volume spectrum



Work at heavier-than-physical light quarks masses:

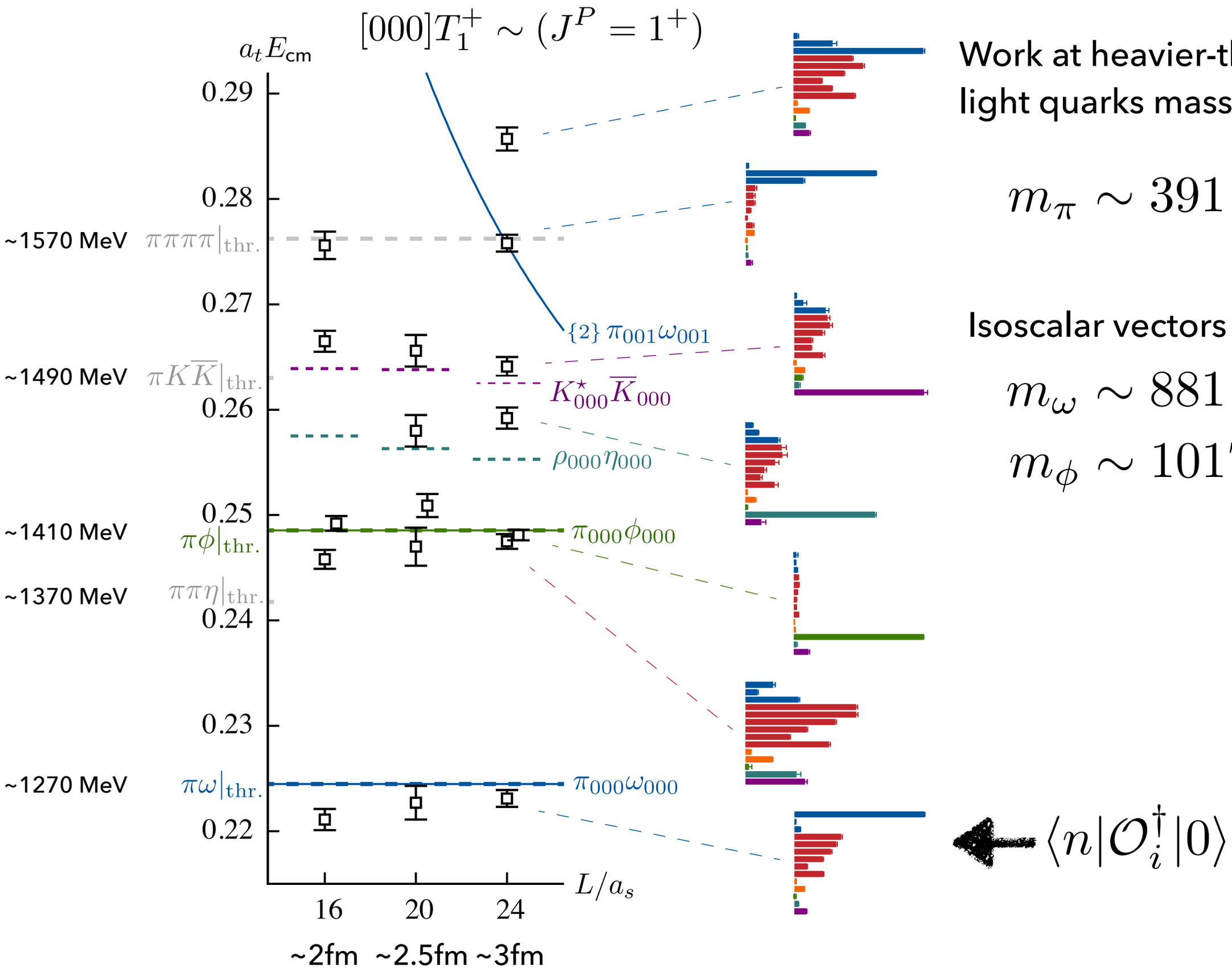
$$m_\pi \sim 391 \text{ MeV}$$

Isoscalar vectors are stable:

$$m_\omega \sim 881 \text{ MeV}$$

$$m_\phi \sim 1017 \text{ MeV}$$

$\langle n|\mathcal{O}_i^\dagger|0\rangle$



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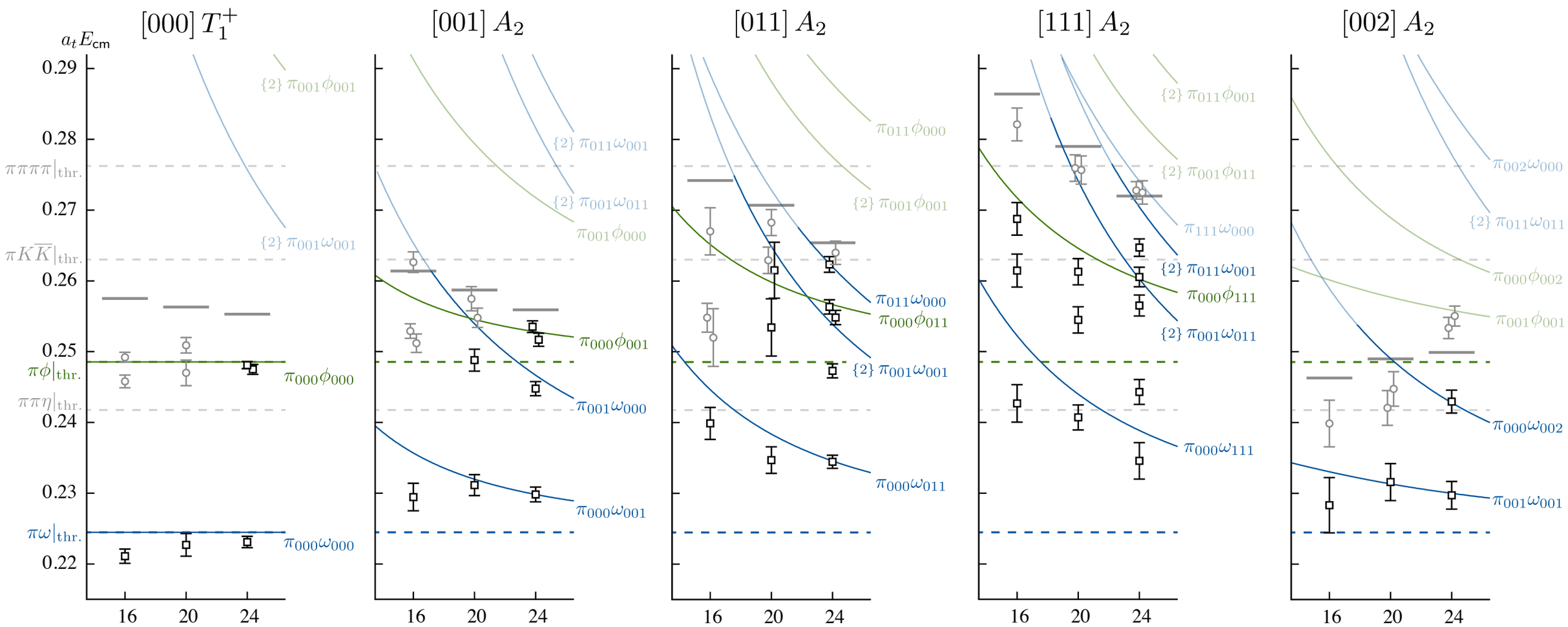
36 lattice energy levels used to constrain the scattering amplitudes



How is it done?

finite-volume spectra → Lüscher's Quantisation Condition → amplitudes

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\left(\mathbf{1} + i\mathbf{M}(E,L)\right)\right] = 0$$

phase-space

scattering matrix

matrix of known finite-volume functions

- difficult problem! parameterise energy dependence of scattering matrix and vary amplitudes until best description of lattice spectra is found

time to look at an explicit calculation…

Work at heavier-than-physical light quarks masses: $m_\pi \sim 391 \text{ MeV}$

- isoscalar vectors are stable $m_\omega \sim 881 \text{ MeV}$

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- consider **three-coupled** meson-meson channels

$\pi\omega\{^3S_1\} \quad \pi\omega\{^3D_1\} \quad \pi\phi\{^3S_1\} \quad \pi\phi\{^3D_1\}$

- OZI suppressed
- centrifugal barrier suppressed

$$J^P = 1^+$$

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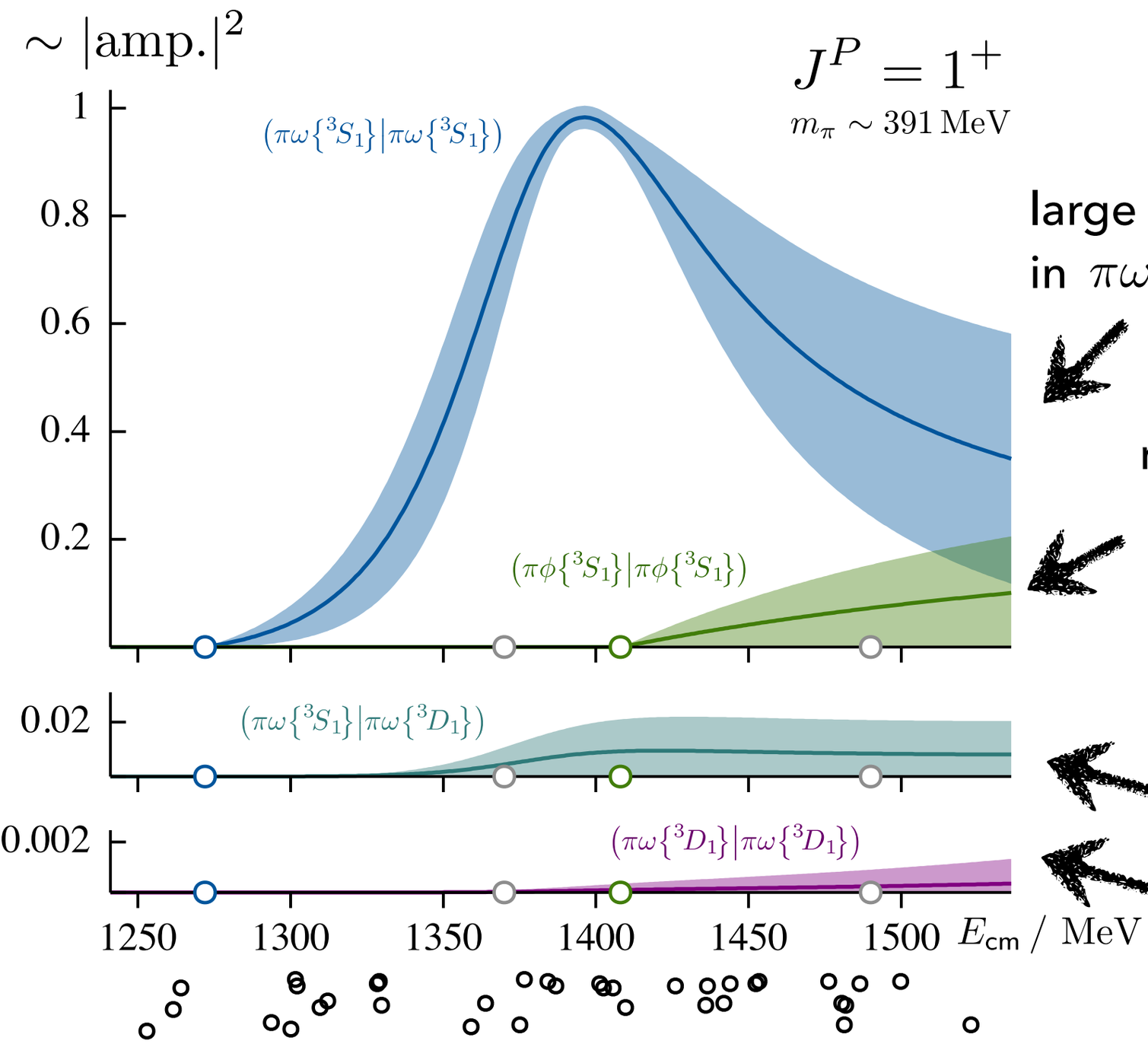
`dynamically-coupled'

coupled hadron-hadron

$$J^P = 1^+$$

$$\mathbf{t} = \begin{pmatrix} t(\pi\omega\{^3S_1\}|\pi\omega\{^3S_1\}) & t(\pi\omega\{^3S_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3S_1\}|\pi\phi\{^3S_1\}) \\ & t(\pi\omega\{^3D_1\}|\pi\omega\{^3D_1\}) & t(\pi\omega\{^3D_1\}|\pi\phi\{^3S_1\}) \\ & & t(\pi\phi\{^3S_1\}|\pi\phi\{^3S_1\}) \end{pmatrix}$$



large canonical `bump' enhancement in $\pi\omega\{^3S_1\} \to \pi\omega\{^3S_1\}$

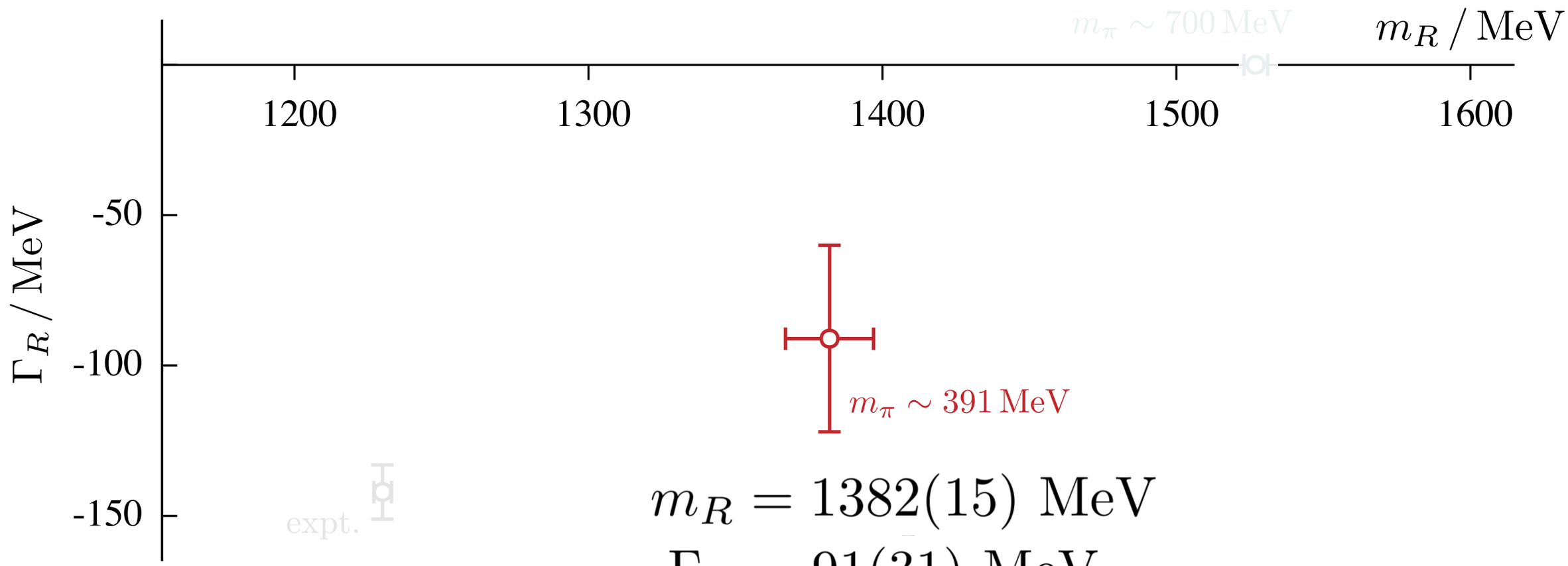
relatively small in $\pi\phi\{^3S_1\} \to \pi\phi\{^3S_1\}$

no evidence of Z_s enhancement proposed as an analogue of the Z_c

small $\pi\omega\{^3S_1\} \to \pi\omega\{^3D_1\}$

negligible $\pi\omega\{^3D_1\} \to \pi\omega\{^3D_1\}$

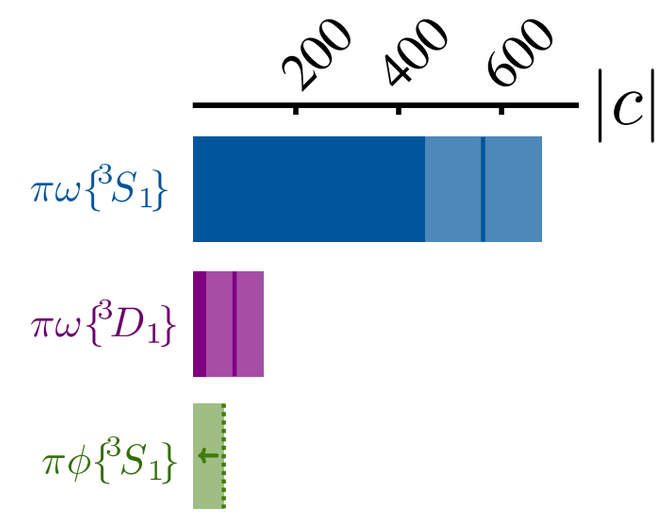
extremely small $\pi\omega \to \pi\phi$ consistent with zero

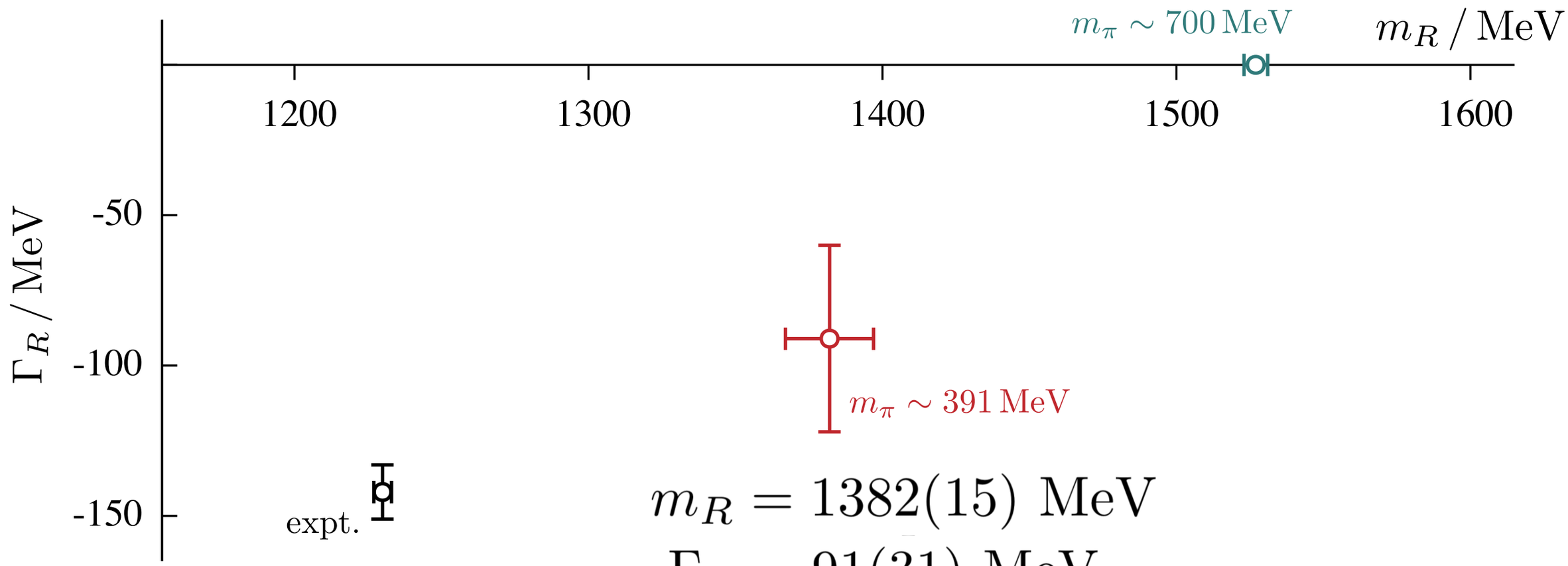


$$m_R = 1382(15) \text{ MeV}$$
$$\Gamma_R = 91(31) \text{ MeV}$$

$$t_{ij}(s) \sim \frac{c_i\, c_j}{s - s_0}$$

$$\left|c_{\pi\omega\{^3S_1\}}\right| = 564(114) \text{ MeV}$$
$$\left|c_{\pi\omega\{^3D_1\}}\right| = 81(56) \text{ MeV},$$
$$\left|c_{\pi\phi\{^3S_1\}}\right| = 59(41) \text{ MeV}.$$





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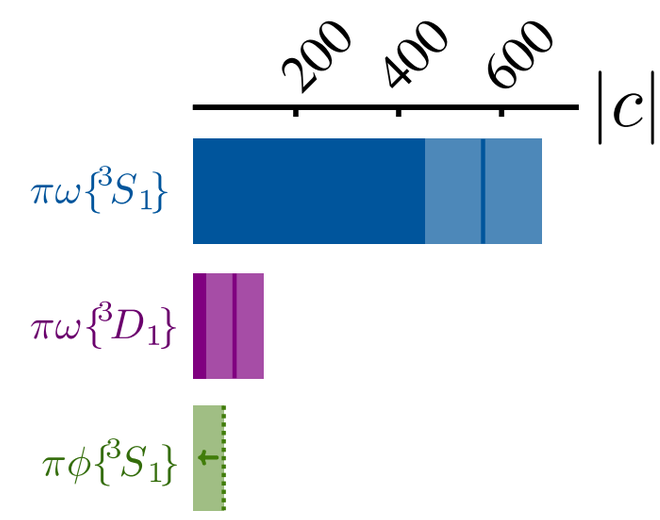


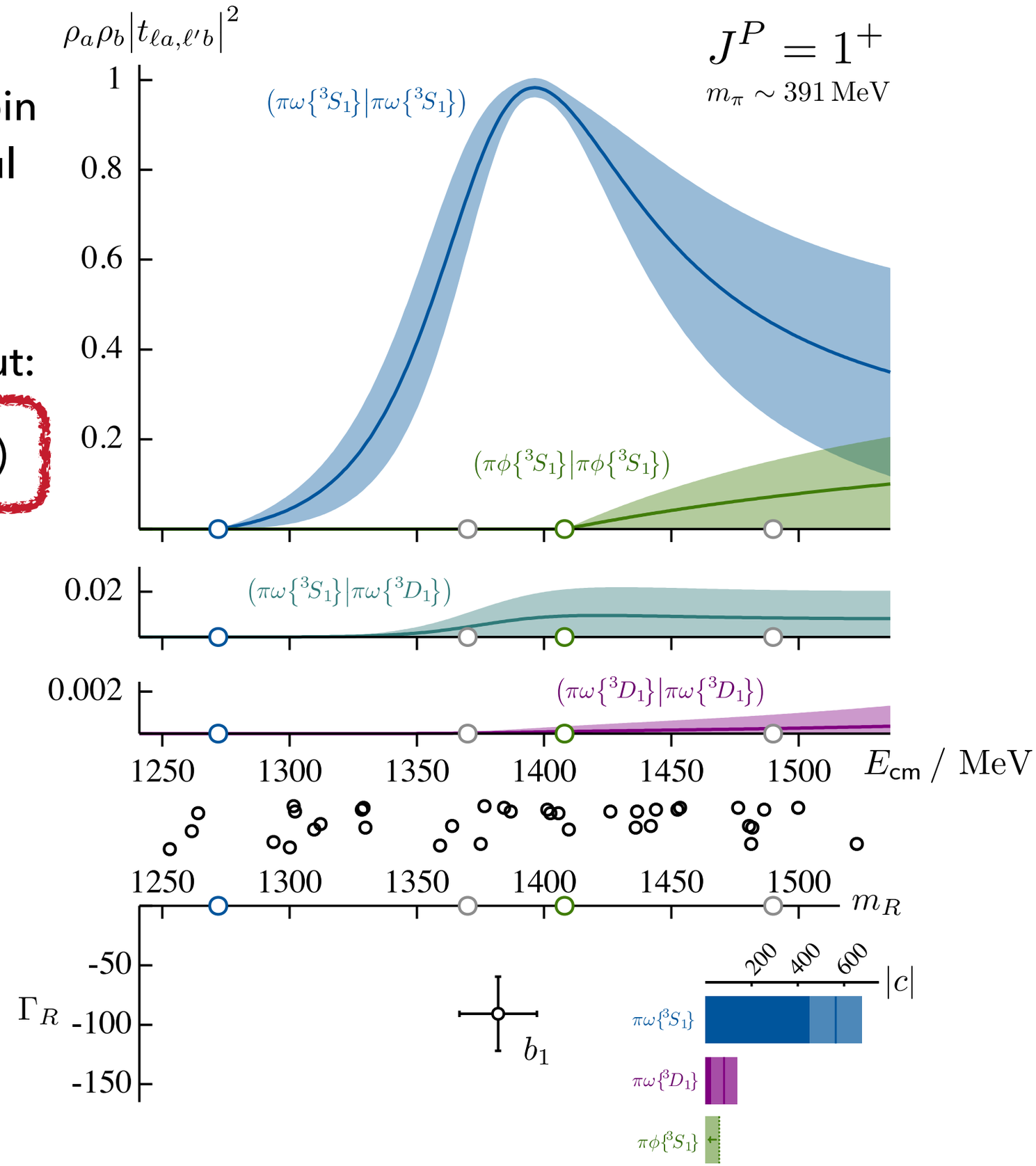
Theory, tools and technology for spin scattering from LQCD very powerful and successful

Many interesting studies to carry out:

$\pi_1(1600)$ ← (finalising!)

$X(3872), Z_c(3900)$

$NN, N\pi$



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THANKS FOR LISTENING