Excited J^{--} meson resonances at the SU(3) flavor point from lattice QCD

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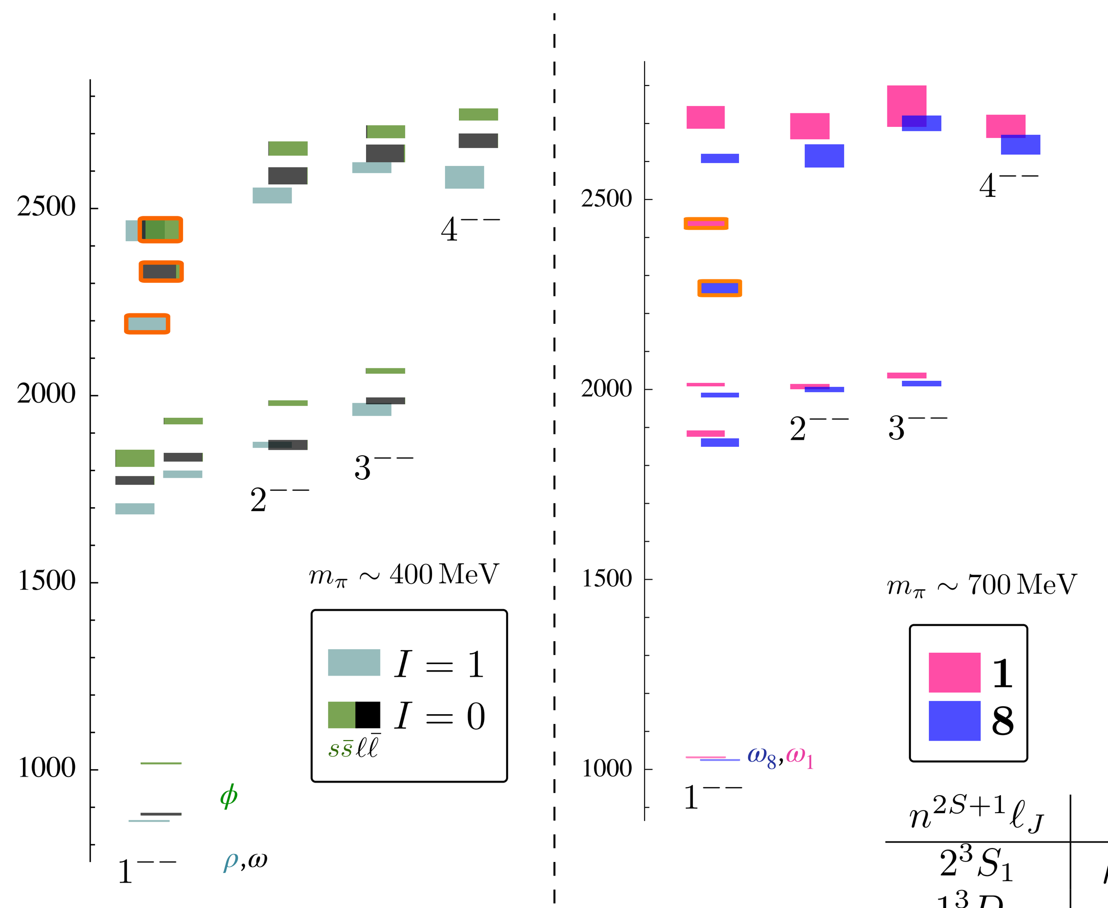
Introduction

Light vector mesons ($J^{PC} = 1^{--}$) ρ, ω, ϕ in the quark model are benchmark resonances in QCD. The similar masses of the ρ and ω, and the preference for ϕ to decay to $K\bar{K}$ final states when there is a much larger phase space for $\pi\pi\pi$, leads to the OZI rule and the $u\bar{d}, u\bar{u}+d\bar{d}, s\bar{s}$ assignment for the three states. Their excited J^{--} resonances are not nearly as well understood due to the many resonances in this region (1-2 GeV), and many decay channels. We study these resonances in scattering where they emerge as intermediate states before decaying into lighter states, using lattice QCD.



(Fig. 1). Lattice QCD calculation [1] predicts there are four J^{--} states in the $1.5 - 2.0$ GeV range for I=1, I=0 (light), and I=0 (strange). Quark content suggests these are excited $\rho_J^*, \omega_J^*, \phi_J^*$ states in the configurations 2^3S_1, $^3D_{1,2,3}$

ℓ	J^P	$n^{2S+1}\ell_J$
0	1^-	n^3S_0
1	$(0,1,2)^+$	$n^3P_{0,1,2}$
2	$(1,2,3)^-$	$n^3D_{1,2,3}$
…		

The different quantum number combinations for a spin S=1 $q\bar{q}$ pair.
n=orbital excitation number
P=parity
ℓ=orbital angular momentum
J=total angular momentum

$n^{2S+1}\ell_J$	ρ_J^*	ω_J^*	ϕ_J^*
2^3S_1	$\rho(1450)$	$\omega(1420)$	$\phi(1680)$
1^3D_1	$\rho(1700)$	$\omega(1650)$	???
1^3D_2	???	???	???
1^3D_3	$\rho_3(1690)$	$\omega_3(1670)$	$\phi_3(1850)$

States confirmed by the PDG with the left column labeling the states as they are classified by the quark model.

SU(3) Flavor

We study these resonances in lattice QCD where we have control over the quark masses. Raising the light quark masses to be equal to the strange quark mass gives rise to a higher SU(3) flavor symmetry and less decay channels. This work studies the SU(3) singlet ω_J^1 excited states (pink bars in Fig. 1).

Physical QCD

$m_\pi \sim 140$ MeV

J^{--} resonances can decay into many final states
SU(2) flavor : $m_u = m_d < m_s$
$\Rightarrow$ π's and ρ's form triplets of indistinguishable particles

$$\begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix} \begin{pmatrix}\rho^+\\ \rho^0\\ \rho^-\end{pmatrix} \Bigg\} \text{Triplet}$$

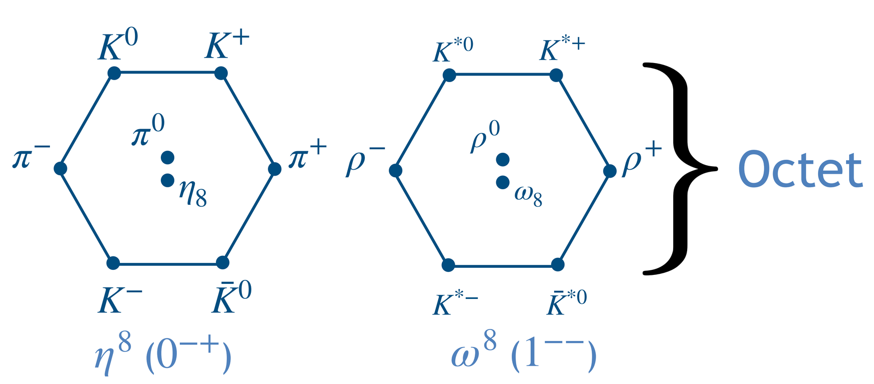
$\pi\ (0^{-+})$ $\rho\ (1^{--})$

$\eta\ (0^{-+})$ $\omega\ (1^{--})$ } Singlet

This calculation

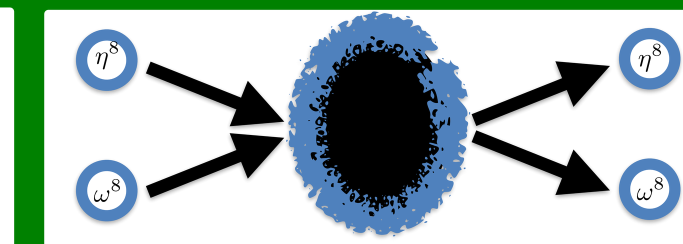
$m_\pi \sim 700$ MeV

J^{--} resonances have much fewer final states
SU(3) flavor : $m_u = m_d = m_s$
$\Rightarrow$ mesons form octets of indistinguishable particles



$\eta^8\ (0^{-+})$ $\omega^8\ (1^{--})$

$\eta^1\ (0^{-+})$ $\omega^1\ (1^{--})$ } Singlet



The J^{--} (ω_J^1) resonances can be studied in $\eta^8\omega^8$ elastic scattering.

We study scattering in lattice QCD by:
1). Computing the energy spectrum on the lattice.
2). Relate the discrete energy levels to the scattering amplitude via a known mapping
3). Search for poles (resonances) in the scattering amplitude.

ℓ	J^P $(\eta^8\omega^8)$
0	1^+
1	$0^-, 1^-, 2^-$
2	$1^+, 2^+, 3^+$
3	$2^-, 3^-, 4^-$
…	…

Relevant partial waves (red) for $\eta^8\omega^8$.

Extracting the energy spectrum

We extract the finite-volume spectrum through by computing a matrix of correlation functions on the lattice featuring a variety of single-meson, and meson-meson type operators that possess J^{--} quantum numbers in the continuum. The energy eigenvalues emerge in the time-dependence:

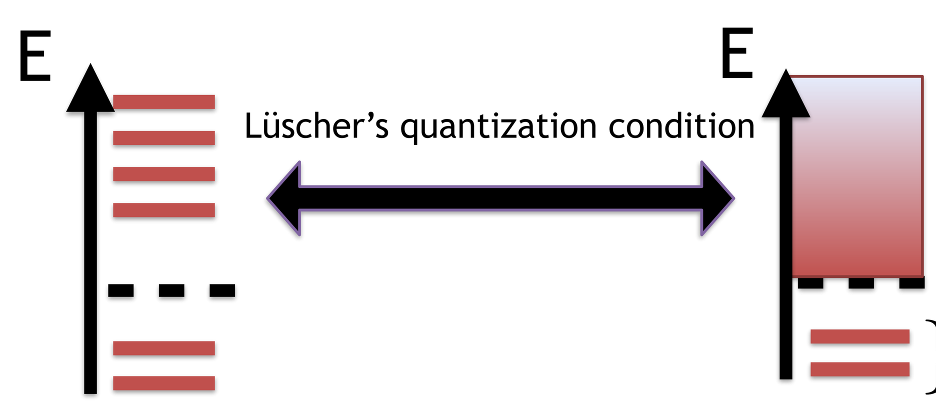
$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0)\rangle = \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle \frac{e^{-E_n t}}{2E_n}$$

Energy eigenvalues can be extracted by solving a generalized eigenvalue problem [1,3], and fitting the time-dependance of the eigenvalues:

$$C(t)v^n = \lambda^n(t) C(t_0) v^n \qquad \lambda^n(t) \sim e^{-E_n(t-t_0)}$$

Finite volume and scattering

The energy levels computed from the lattice are directly related to the scattering amplitude in an infinite volume.



Energy levels of a system in a finite periodic box are discrete, unlike the continuous spectrum above threshold one expects in scattering.There exists a relation between the finite-volume spectrum and the scattering amplitude known as Lüsher's quantization condition [2].

$$\det\Big[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \underbrace{\boldsymbol{t}(E)}_{\text{Scattering amplitude}} \cdot \big(\mathbf{1} + i\underbrace{\boldsymbol{\mathcal{M}}(E,L)}_{\text{Known matrix of finite-volume functions}}\big)\Big] = 0$$

Each volume provides different energy eigenvalues. We extract energy levels on 5 volumes L=14,16,18,20,24 to constrain the energy-volume relationship.

We parameterize the scattering amplitude using the K-matrix so that we satisfy unitarity (probability conservation) and fit to the data:

$$\mathbf{t}^{-1}_{\ell\ell'}(E) = \left(\frac{1}{2k}\right)^{\ell} \mathbf{K}_{\ell\ell'}(E) \left(\frac{1}{2k}\right)^{\ell'} - i\boldsymbol{\rho}(E)\delta_{\ell\ell'}$$

Unitarity constrains the imaginary part to be the phase space = 2k/E

$$K_{J=3} = \frac{g^2}{(m_R^{J=3})^2 - s}$$

F-wave resonance represented with a pole term.

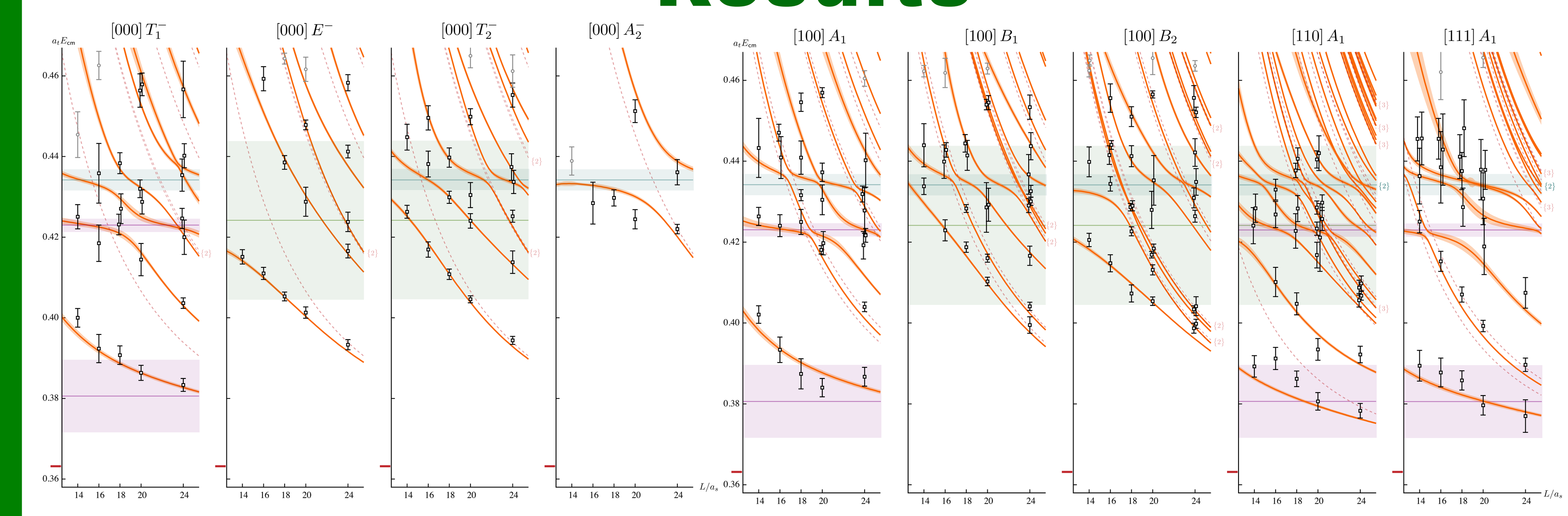
$$K_{J=2} = \begin{pmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{pmatrix} \frac{1}{(m_R^{J=2})^2 - s} + \begin{pmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & \gamma_{FF} \end{pmatrix}$$

P- and F-waves both contribute to the 2^{--} amplitude in the form of a pole, and matrix of constants.

$$K_{J=1} = \frac{g_a^2}{(m_a^{J=1})^2 - s} + \frac{g_b^2}{(m_b^{J=1})^2 - s} + \gamma s^n$$

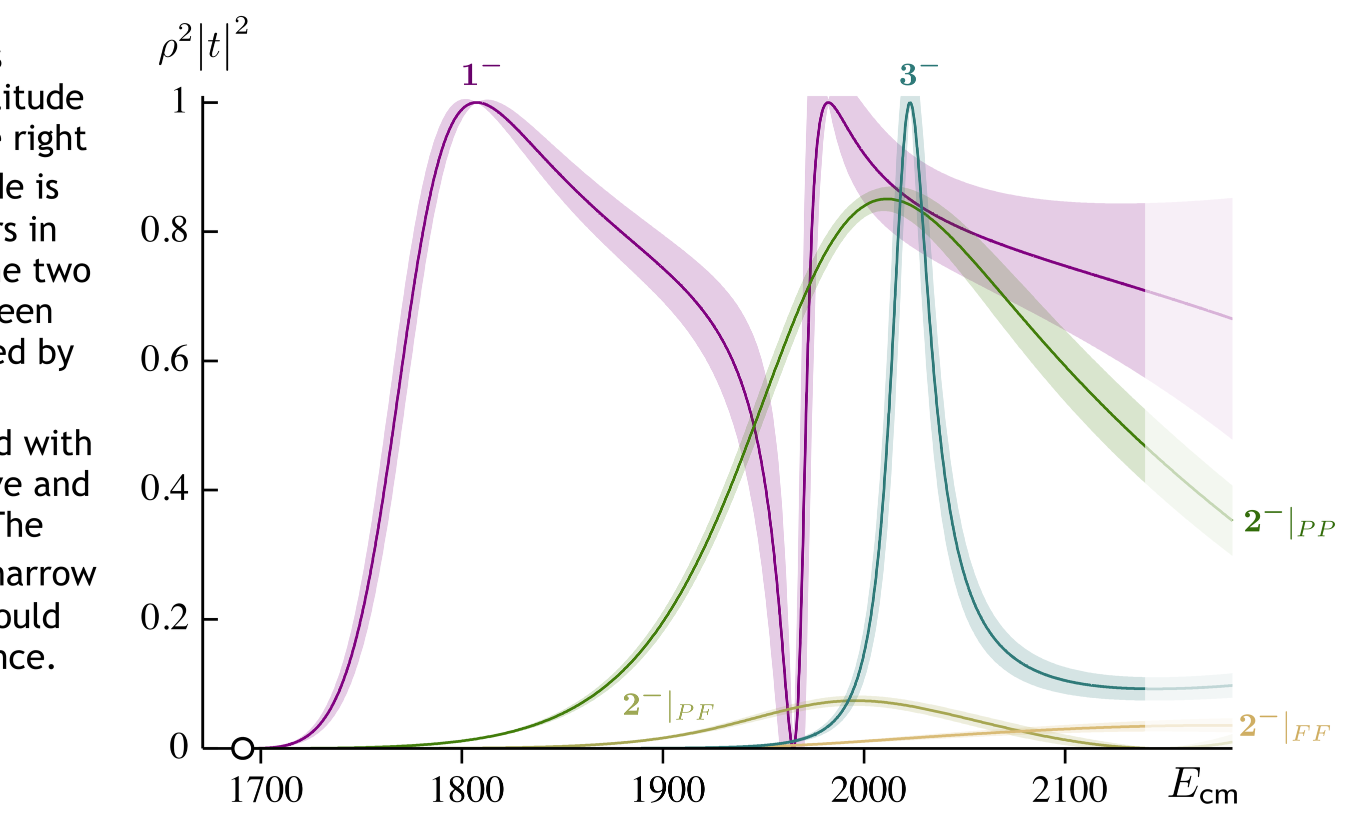
Two poles are used to parameterize the two resonances in this partial wave.

Results



(Fig. 2). Finite-volume spectrum in black points with the amplitude parameterization represented by the orange curves. The purple (1^{--}), green (2^{--}), and cyan (3^{--}) show the masses and widths of the resonances.

Resonances usually appear as *bumps* in the scattering amplitude which are represented to the right as $\rho^2|t|^2$. The 1^{--} amplitude is very different from the others in that it has a zero between the two peaks. This zero that is between the two resonances is required by elastic unitarity. The 2^{--} amplitudes appear very broad with a large coupling to the P-wave and a small coupling to F-wave. The 3^{--}amplitude appears very narrow characteristic of what one would expect for an F-wave resonance.

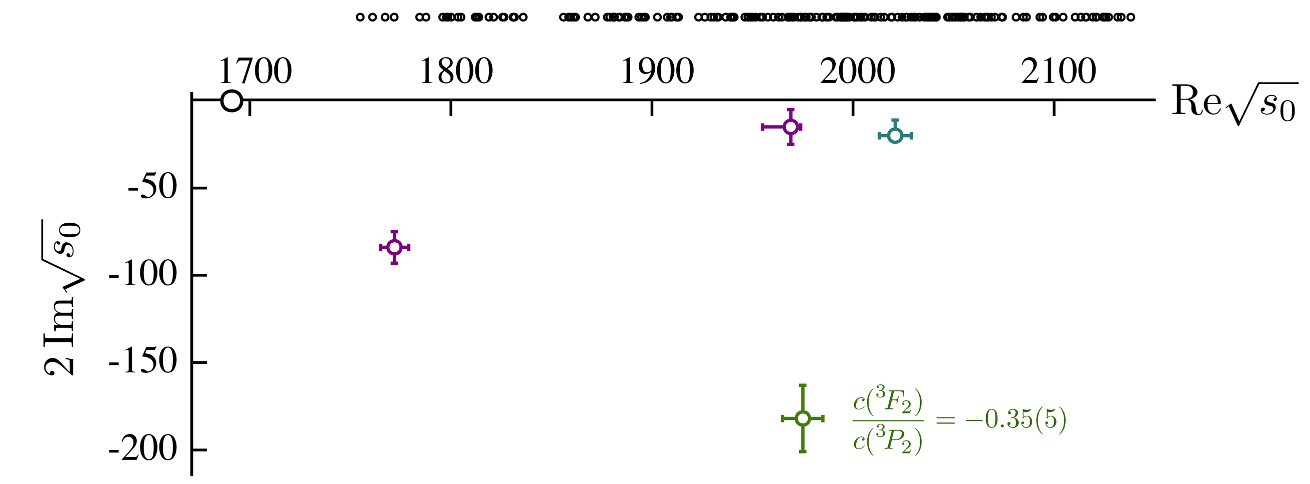


Resonances are mathematically understood as pole singularities in $s = E^2$ of the scattering amplitude:

$$t(s) \sim \frac{\tilde{t}(s)}{s_0 - s}$$

The real part of the pole is related to the mass, and the imaginary part the width:

$$\sqrt{s_0} = m_R \pm \frac{i}{2}\Gamma$$



(Fig. 3). Pole positions of the 4 resonances displayed under the amplitudes. The masses are the horizontal axis and the widths are the magnitude of the vertical axis.

Conclusion

We report 4 resonances in the singlet representation of SU(3) flavor, a lighter broad state followed by a heavier narrow state in 1^{--}, a broad 2^{--} state coupling mostly to P-wave, and a narrow 3^{--} state. This supports quark model expectations of a lighter 2^3S_1 resonance, and three nearly mass degenerate $^3D_{1,2,3}$ resonances. Future work will determine the resonances in the octet representation ω_J^8 in coupled channel scattering.

References

[1] J.J. Dudek, R.G. Edwards, P. Guo, and C.E. Thomas (Hadron Spectrum), Phys. Rev. **D88**, 094505 (2013)
[2] M. Lüscher, Nucl. Phys. **B354**, 531 (1991)
[3] C. Michael, Nucl. Phys. **B259**, 58 (1985)

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