Ivan Vitev

# Effective field theories in reactions with nuclei: from jets to quarkonia

Based on: ArXiv: 1811.07905, 1903.06170 , 1907.04419, 1908.06979 With: Haitao Li, Yiannis Makris, Matt Sievert, Boram Yoon

Nuclear Theory Seminar, JLab, Agust 2019

Newport News, VA



### **Outline of the talk**

- A brief introduction to effective field theories (EFTs)
- An effective theory for jet propagation in matter - SCET<sub>G</sub> Current status and in-medium splitting functions to arbitrary order in opacity
- Application of SCET<sub>G</sub> to heavy ion phenomenology – b-jets and the jet charge
- An effective theory of qaurkonia in matter – NROCD<sub>G</sub>. Inapplicability of the energy loss approach to current measurements
- Derivation of the leading order and next to leading order NROCD<sub>G</sub> Lagrangian using different methods
- Connection to quarkonium dissociation in matter and existing phenomenology
- Conclusions



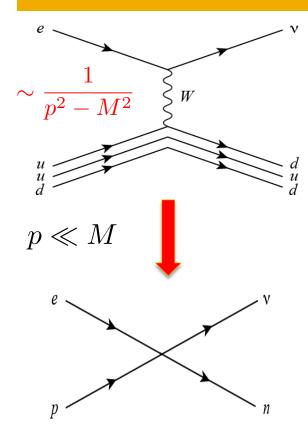
Thanks for the invitation!

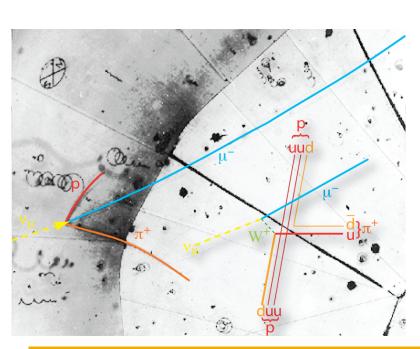
### Introduction



#### **The Fermi interaction**

 The first, probably best known, effective theory is the Fermi interaction



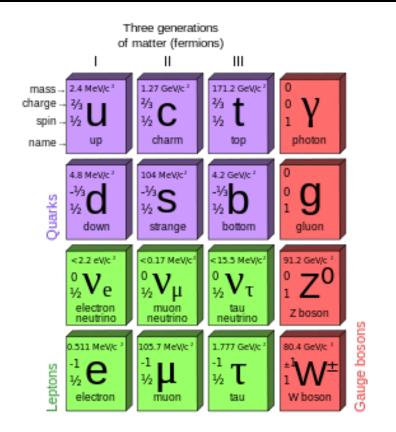




E. Fermi (Nobel Prize)

 First direct observation of the neutrino, Nov. 1970

#### **Effective field theories**



- Powerful framework based on exploiting symmetries and controlled expansions for problems with a natural separation of energy/momentum or distance scales.
- Particularly well suited to QCD and nuclear physics
- Effective theories are ubiquitous. The Standard Model is likely a low energy EFT of a theory at a much higher scale

### Examples of effective field theories [EFTs]

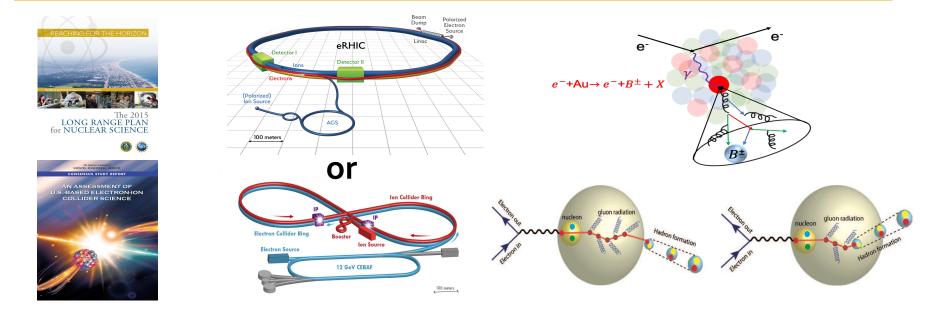
DOF in FT	E Full Theory Effective	<ul> <li>Focus on the significant degrees of freedom [DOF]. Manifest power counting</li> </ul>			
	Theory	Q power counting DOF in FT DOF in EFT			
Chiral Perturb	Nqcd	p/Aqcd	q, g	К, <b>π</b>	
Heavy Quark Effective Theory (HQET)		mь	NQCD/Mb	ψ,Α	hv,As
Soft Collinear Effective Theory (SCET)		Q	p⊥/Q	ψ,Α	ξn,An,As
Non-Relativistic QCD (NRQCD)		m <sub>Q</sub>	p/m <sub>Q</sub>	ψ,Α	Ψ <sub>Q</sub> , As, Aus

## Soft Collinear Effective Theory with Glauber gluons



#### Motivation

 Jet and heavy flavor production in reactions with nuclei is an essential part of modern collider physics. Also at the future electron ion collider. New insights into the transport of energy and matter through a strongly-interacting quantum-mechanical environment

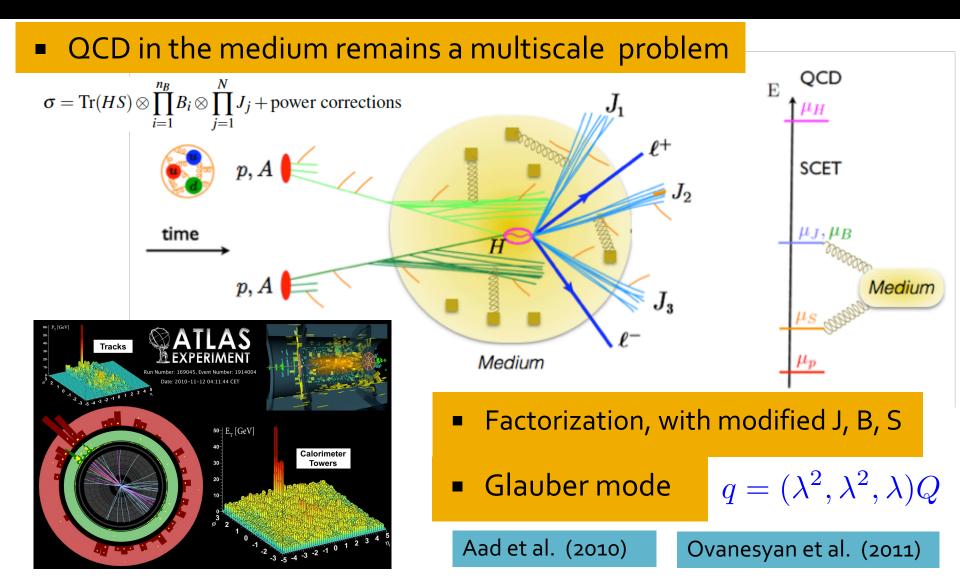


- In the meantime jets and heavy flavor are the bread and butter physics at RHIC and LHC.
- Enormous amount of data exists and new measurements continue

### **SCET formulation**

Modes in SCET	C. Bauer e	C. Bauer et al. (2001)		D. Pirol et al. (2004)	
			M. Benek	(e et al. (2004)	
Collinear quarks, antiquarks	$\xi_n, \ \overline{\xi}_n$	Soft quai	rks are elimin	ated through	
Collinear gluons, soft gluons	$A_n, A_s$	the equations of motion		5	
SCET <sub>II</sub>	modes collinear soft	$p^{\mu} = (+, -, +)$ $Q(\lambda^2, 1, +)$ $Q(\lambda, \lambda, \lambda)$	$\lambda) = Q^2 \lambda^2$		
<ul> <li>Other formulations, e.g. SCET<sub>I</sub> and ultrasoft particles</li> </ul>					
Especially	$\sigma = \mathrm{Tr}(HS) \otimes \prod_{i=1}^{n_B} E$	$B_i \otimes \prod_{j=1}^N J_j + \text{power corr}$	rections		
suited for jet	δ linear t			δ n-collinear jet	

### The big picture in QCD matter



#### The Glauber gluon Lagrangian

An effective theory of jet propagation in matter

Effective potential

Soft source

Static source

$$\mathcal{L}_{\mathcal{G}}\left(\xi_{n}, A_{n}, \eta\right) = \sum_{p, p', q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n, p'} \Gamma^{\mu, a}_{qqA_{\mathcal{G}}} \frac{\vec{\eta}}{2} \xi_{n, p} - i \Gamma^{\mu\nu\lambda, abc}_{ggA_{\mathcal{G}}} \left(A^{c}_{n, p'}\right)_{\lambda} \left(A^{b}_{n, p}\right)_{\nu} \right) \bar{\eta} \Gamma^{\delta, a}_{\mathbf{s}} \eta \, \Delta_{\mu\delta}(q)$$

$$A. \text{ Idilbi et al. (2008)}$$

Gauge

Object

 Feynman rules for different sources and gauges

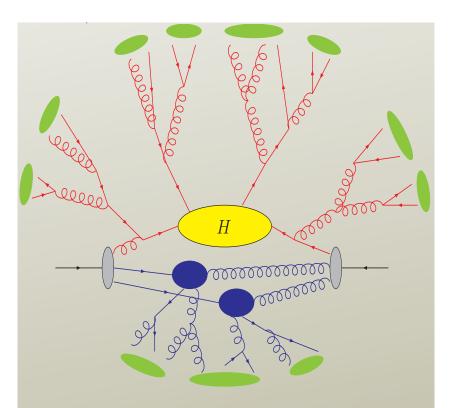
	<i>p</i> .	$[\lambda^2, 1, \boldsymbol{\lambda}]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, \boldsymbol{\lambda}]$
	$a_{\mathbf{p}}, a_{\mathbf{p}}^{\dagger}$	$\lambda^{-1}$	$\lambda^{-3/2}$	$\lambda^{-3/2}$
	u(p)	1	1	$\lambda^{1/2}$
	$\bar{u}(p_2)\gamma_{\nu}u(p_1)$	$\left[\lambda^2,1,oldsymbol{\lambda} ight]$	$[1, 1, \boldsymbol{\lambda}]$	$[\lambda, \lambda, oldsymbol{\lambda}]$
$R_{\xi}$	$A^{\mu}(x)$	$\left[\lambda^4,\lambda^2,oldsymbol{\lambda}^3 ight]$	$[\lambda^2, \lambda^2, \boldsymbol{\lambda}^3]$	$[\lambda, \lambda, \lambda]$
	$\Gamma_{qqA_G}$	$\Gamma_1^{\mu}$	$\Gamma_1^{\mu}$	$\Gamma_{1}^{\mu}$
	$\Gamma_{ggA_G}$	$\Sigma_1^{\mu\nu\lambda}$	$\Sigma_1^{\mu\nu\lambda}$	$\Sigma_1^{\mu\nu\lambda}$
	$\Gamma_{s}$	$\Gamma_1^{\mu} (n \leftrightarrow \bar{n})$	$\Gamma^{\mu}_{3}$	$\Gamma^{\mu}_4$
$A^{+} = 0$	$A^{\mu}(x)$	$\left[0,\lambda^2,oldsymbol{\lambda}^3 ight]$	$\left[0,\lambda^2,oldsymbol{\lambda} ight]$	$[0, \lambda, 1]$
	$\Gamma_{qqA_G}$	$\Gamma_1^{\mu}$	$\Gamma_1^{\mu} + \Gamma_2^{\mu}$	$\Gamma_1^{\mu} + \Gamma_2^{\mu}$
	$\Gamma_{ggA_G}$	$\Sigma_2^{\mu\nu\lambda}$	$\Sigma_2^{\mu\nu\lambda}$	$\Sigma_2^{\mu\nu\lambda}$
	$\Gamma_{s}$	$\Gamma^{\mu}_{2} \left( n \leftrightarrow \bar{n} \right)$	$\Gamma_3^{\mu}$	$\Gamma_4^{\mu}$
	1 1	1		

Collinear source

$$\Sigma_{1}^{\mu\nu\lambda,abc} = gf^{abc} n^{\mu} \left[ g^{\nu\lambda} \bar{n} \cdot p + \bar{n}^{\nu} \left( p_{\perp}^{\prime\lambda} - p_{\perp}^{\lambda} \right) - \bar{n}^{\lambda} \left( p_{\perp}^{\prime\nu} - p_{\perp}^{\nu} \right) - \frac{1 - \frac{1}{\xi}}{2} \left( \bar{n}^{\lambda} p^{\nu} + \bar{n}^{\nu} p^{\prime\lambda} \right) \right] , \qquad (2)$$

G. Ovanesyan et al. (2011)

## The QCD splitting kernels in the vacuum and medium



- In the description of high energy processed significant effort has been devoted to understand the logs, legs and loops
- Splitting functions are related to beam (B) and jet (J) functions in SCET
- Higher order calculations
- Resummation
- Paton showers in Monte Carlos

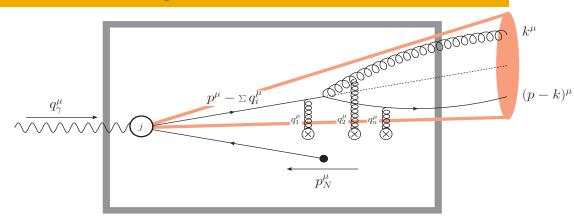
Gribov et al. (1972) G. Altarelli et al. (1977) Y. Dokshitzer (1977) Ovanesyan et al. (2012)

Kang et al. (2016)

 Derivation of splitting kernels to first order in opacity. How about higher orders?

## Theoretical framework for higher orders in opacity

- The theoretical framework is completely general – it is applicable for both cold nuclear matter and the QGP
- This is achieved by isolating the medium in transport parameters and universal gluon-mediated interactions



- The limit we are interested in
- We neglect collisional energy losses

$$\mathcal{L}_{opac.} = \mathcal{L}_{QCD} + \mathcal{L}_{ext}^{qG} + \mathcal{L}_{ext}^{gG} + \mathcal{L}_{G.F.} + \cdots$$
$$v(q_T^2) \rightarrow \frac{-g_{eff}^2}{q_T^2 + \mu^2} \qquad \frac{d\sigma^{el}}{d^2q} = \frac{1}{(2\pi)^2} \frac{C_F}{2N_c} [v(q_T^2)]^2$$

In deep inelastic scattering (DIS) the lowest order processes involve prompt quark. Even at NLO the prompt gluon jet contribution is small

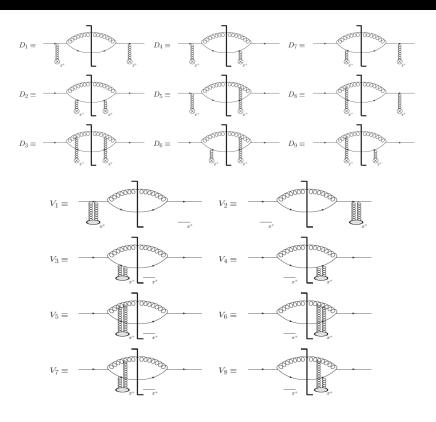
F. Ringer et al . (2018)  $\frac{1}{p_N^-} \ll l_f^+ \sim \lambda^+ \sim L^+$   $\mathcal{O}\left(\frac{\perp^2}{Q^2}\right)$ 

## Lightcone wave functions and parton branchings

$$xp^{+} \left. \frac{dN}{d^{2}k \, dx \, d^{2}p \, dp^{+}} \right|_{\mathcal{O}(\chi^{0})} = \frac{\alpha_{s} \, C_{F}}{2\pi^{2}} \, \frac{(k-xp)_{T}^{2} \, \left[1+(1-x)^{2}\right]+x^{4}m^{2}}{[(k-xp)_{T}^{2}+x^{2}m^{2}]^{2}} \, \times \left(p^{+} \frac{dN_{0}}{d^{2}p \, dp^{+}}\right)$$

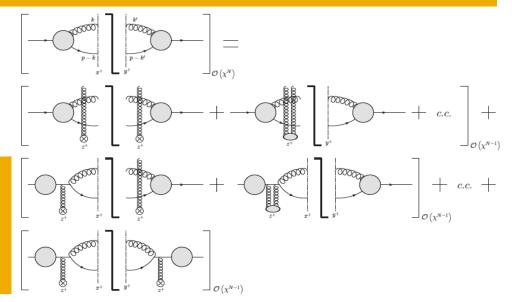
- Certain advantages can provide in "one shot" both massive and massless splitting functions
- Have checked that results agree for massless and massive DGLAP

## Opacity expansion – direct and virtual diagrams



 Finally, relative to the splitting vertex we classify them as
 Initial/Initial, Initial/Final, Final/Initial and Final/Final

- Interaction in the amplitude and the conjugate amplitude (Direct or single Born diagrams)
- Interaction in the amplitude or the conjugate amplitude (Virtual or double Born diagrams)
- Use them to calculate both the medium evolution kernels & initial conditions



#### Master equation – matrix form

 Upper triangular structure. Suggests specific strategy how to solve it. Calculated: initial conditions, kernels, and wrote a Mathematica code to solve it

$$\begin{bmatrix} f_{F/F}^{(N)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{F/I}^{(N)}(\underline{p}; x^+, y^+) \end{bmatrix} = \int_{x_0^+}^{\min[x^+, y^+, R^+]} \frac{dz^+}{\lambda^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \begin{bmatrix} \mathcal{K}_1 \ \mathcal{K}_2 \ \mathcal{K}_3 \ \mathcal{K}_4 \\ 0 \ \mathcal{K}_5 \ 0 \ \mathcal{K}_6 \\ 0 \ 0 \ \mathcal{K}_7 \ \mathcal{K}_8 \\ 0 \ 0 \ 0 \ \mathcal{K}_9 \end{bmatrix} \begin{bmatrix} f_{F/F}^{(N-1)}(\underline{k}, \underline{k}', \underline{p}; x^+, y^+) \\ f_{I/F}^{(N-1)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{F/I}^{(N-1)}(\underline{k}, \underline{p}; x^+, y^+) \\ f_{I/I}^{(N-1)}(\underline{p}; x^+, y^+) \end{bmatrix}$$

#### Simplest kernel

$$\mathcal{K}_{9} = \left[ e^{-\underline{q} \cdot \underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] + \left[ -\frac{1}{2} \right] \left[ e^{+(z^{+}-x^{+})\partial_{x^{+}}} + e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right]$$

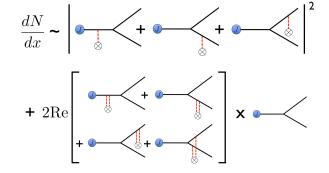
#### Most complicated kernel

$$\begin{split} \mathcal{K}_{1} &= \left[ e^{i[\Delta E^{-}(\underline{k}-x\underline{p}+x\underline{q})-\Delta E^{-}(\underline{k}-x\underline{p})]z^{+}} e^{i[\Delta E^{-}(\underline{k}'-x\underline{p})-\Delta E^{-}(\underline{k}'-x\underline{p}+x\underline{q})]z^{+}} \right] \left[ e^{-\underline{q}\cdot\underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] \\ &+ \left[ \frac{N_{c}}{C_{F}} e^{i[\Delta E^{-}(\underline{k}-x\underline{p}-(1-x)\underline{q})-\Delta E^{-}(\underline{k}-x\underline{p})]z^{+}} e^{i[\Delta E^{-}(\underline{k}'-x\underline{p})-\Delta E^{-}(\underline{k}'-x\underline{p}-(1-x)\underline{q})]z^{+}} \right] \\ &\times \left[ e^{-\underline{q}\cdot\underline{\nabla}_{k}} e^{-\underline{q}\cdot\underline{\nabla}_{k'}} e^{-\underline{q}\cdot\underline{\nabla}_{p}} e^{+(z^{+}-x^{+})\partial_{x^{+}}} e^{+(z^{+}-y^{+})\partial_{y^{+}}} \right] \\ &+ 8 \text{ more lines} \end{split}$$

## Generalizing the result to all in-medium splittings (4)

Note – all splittings have the same topology.
 Same - structure, interference phases,
 propagators

Different - mass dependence, wavefunctions, color (which also affects transport coefficients)



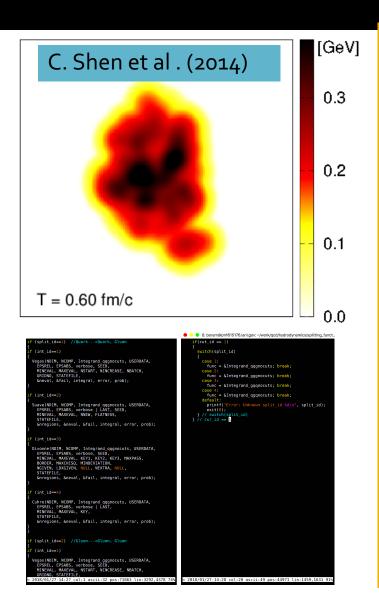
$$\langle \psi(x,\underline{\kappa})\,\psi^*(x,\underline{\kappa}')\rangle = \frac{8\pi\alpha_s\,f(x)}{[\kappa_T^2 + \nu^2 m^2]\,[\kappa_T'^2 + \nu^2 m^2]} \left[g(x)\,(\underline{\kappa}\cdot\underline{\kappa}') + \nu^4 m^2\right] \ \Delta E^-(\underline{\kappa}) = -\frac{\kappa_T^2 + \nu^2 m^2}{2x(1-x)p^+}$$

#### Master table that gives all ingredients

We have now solved the problem for all splitting functions. Answers exact but lengthy – 2<sup>nd</sup> order ~ 10 pages

M. Sievert et al . (2019)

### Improvements in physics & code



#### Refactoring

➤ Code is restructured (in C++) and shortened (24K → 8K lines). 20x speed improvement

#### Effective incorporation of simulated QGP medium

Reduced overhead for calling QGP medium grid function. 2x speed improvement

#### **Efficient on-node parallelization**

New parallelization shows much better scaling 10x speed improvement

Overall improvement: **18 days** → **1 hour** 

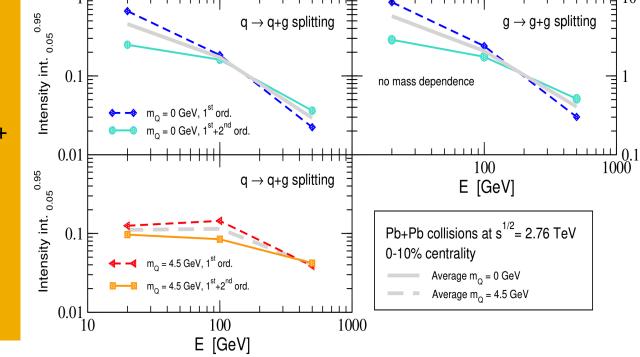
## Medium-induced splitting intensity

#### Porting to code

 Results are directly exported from Mathematica to C++

#### Challenges

 Arise from larger number of evaluations



 $\mathcal{I}_{x_{\min}}^{x_{\max}} = \int_{x_{\min}}^{x_{\max}} dx \int d^2k \ x \frac{dN}{d^2k \ dx}$ 

Energy loss – not a well defined concept for parton shower processes - define splitting intensity

 The main result is a change in the energy dependence of the splitting intensity – smother, or more slowly varying with E (understand jet modification with p<sub>T</sub>)

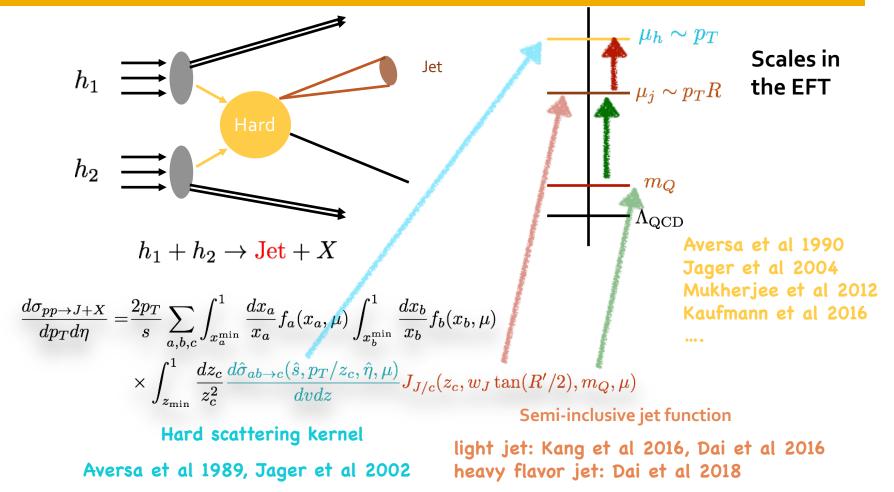
## Phenomenological applications of SCET<sub>G</sub>



"I'm firmly convinced that behind every great man is a great computer."

### Inclusive heavy jet production

• Jet production is one of the cornerstone processes of QCD. Light jets have been studied for a long time. Recent advances for heavy jets (e.g. b) based in SCET



## Resummation

- Evolution between the jet scale and the hard scale hadle by DGLAP evolution of the SIJF
- Evolution from the heavy quark mass to the jet scale is separated into

The SiJFs Evolve according to DGLAP-like equations

$$\frac{d}{d\ln\mu^2} \left( \begin{array}{c} J_{J_Q/s}(x,\mu) \\ J_{J_s/g}(x,\mu) \end{array} \right) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left( \begin{array}{cc} P_{qq}(z) & 2P_{gq(z)} \\ P_{qg}(z) & P_{gg}(z) \end{array} \right) \left( \begin{array}{c} J_{J_Q/s}(x/z,\mu) \\ J_{J_s/g}(x/z,\mu) \end{array} \right)$$

We use the Mellin moment space approach to solve this equation

Resums In  $\mu/p_T R$ 

scales

 $\mu_j \sim p_T h$ 

 $m_O$ 

 $\Lambda_{\rm QCD}$ 

 $\ln R$ 

m

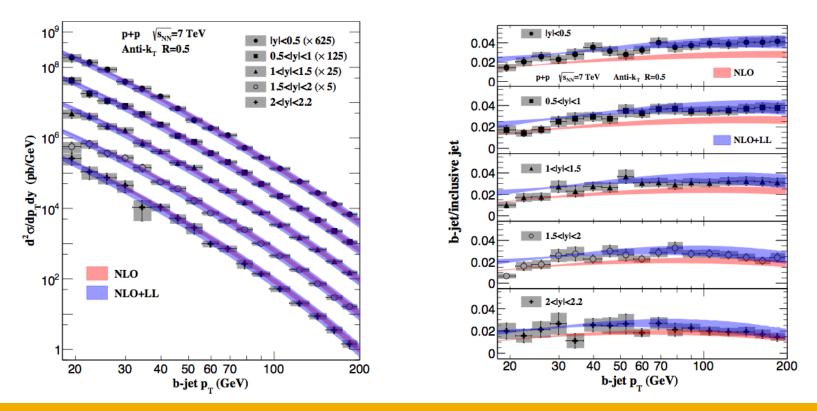
 $p_T R$ 

ln -

$$\mathcal{M}_{g \to Q\bar{Q}}^{\text{in-jet}}(p_T R, m) = 2 \sum_{l=g,Q} \bar{K}_{l/g}(p_T R, m, \mu_F) \bar{D}_{Q/l}(m, \mu_F)$$
Resums ln p<sub>T</sub>R/m  
The integrated perturbative kernel at the jet typical scale The integrated parton fragmentation function from parton *l* to parton *Q*

Bauer, Mereghetti 2013, Dai, Kim, Leibovich 2016, 2018

### B-jet production in pp collisions



Data are consistent with the theoretical predictions

For the ratio b-jets to inclusive jets the difference between NLO+LL and NLO can be traced also to the differences in the inclusive jet cross section

#### **Corrections in A+A collisions**

Let us now focus on the jet function and final-state modification in the QGP

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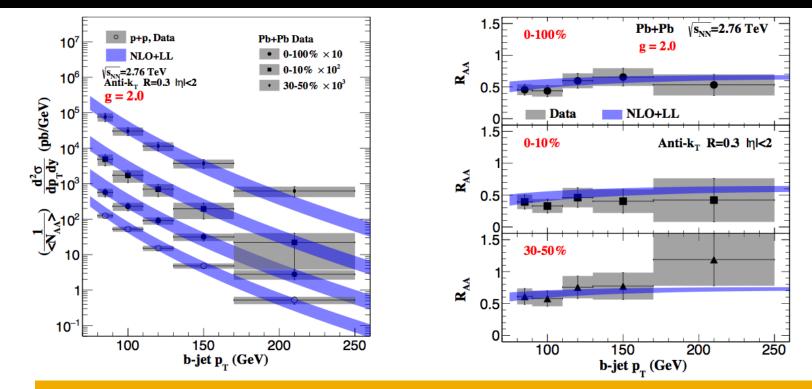
 $\mathcal{O}(\alpha_s \times \frac{L}{\lambda})$ 

 $\mathcal{O}(\alpha_s^0 \times \frac{L}{\lambda})$ 

 Medium induced corrections to the LO jet function

 Medium induced corrections to the NLO jet function

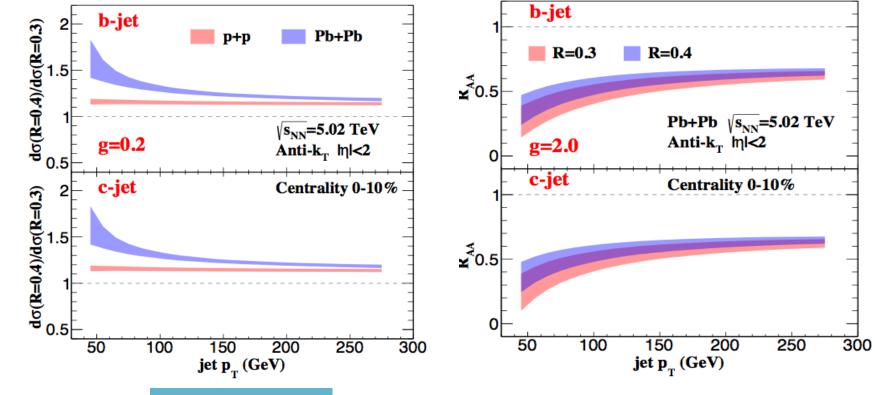
## B-jet production in A-A collisions



- Slightly less dependence on the centrality when compared to the well-known light jet modification
- Theoretical results agree well with the data for both the inclusive cross sections and the nuclear modification factors

#### Does not mean there is no room for improvement. Extend to lower $p_T$

### B-jet and c-jet production in A-A collisions



Haitao Li, Vitev, 2018

- The smaller radius jet tends to dissipate more energy in the medium
- No significant difference between the c-jet and b-jet due to the high transverse momentum
- Not depend on jet p<sub>T</sub> in p+p collisions
- Small dependence on jet p⊤ in Pb+Pb collisions

## The jet charge

- Weighted sum of the charges of all particles in a jet. Proxy for the charge of the quark or gluon
- Allows for jet flavor separation (upquark vs down quark) quark-antiquark separation. Modern machine learning techniques
   K. Fraser *et al. (2018)*

$$\langle Q_{\kappa,q} \rangle = \int dz \ z^{\kappa} \sum_{h} Q_{h} \frac{1}{\sigma_{q\text{-jet}}} \frac{d\sigma_{h\in q\text{-jet}}}{dz}$$
$$\langle Q_{\kappa,q} \rangle = \frac{\tilde{\mathcal{J}}_{qq}(E,R,\kappa,\mu)}{J_{q}(E,R,\mu)} \tilde{D}_{q}^{Q}(\kappa,\mu)$$

 Expressed in (k+1) Mellin moment of the jet matching coefficient and charge-weighted frag. function

$$\begin{split} \tilde{\mathcal{J}}_{qq}(E,R,\kappa,\mu) &= \int_0^1 dz \ z^{\kappa} \mathcal{J}_{qq}(E,R,z,\mu) \ , \\ \tilde{D}_q^Q(\kappa,\mu) &= \int_0^1 dz \ z^{\kappa} \sum_h Q_h D_q^h(z,\mu) \end{split}$$

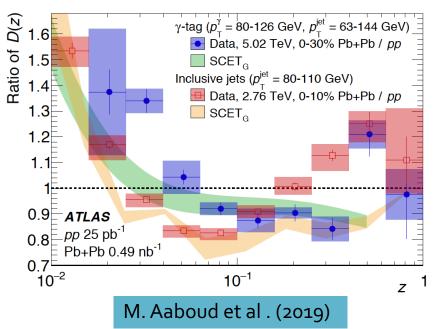
$$Q_{\kappa,\text{jet}} = \frac{1}{\left(p_T^{\text{jet}}\right)^{\kappa}} \sum_{i \in \text{jet}} Q_i \left(p_T^i\right)^{\kappa} \qquad \text{R. Field et al. (1978)}$$

$$J. \text{ Berge et al. (1981)} \qquad J. \text{ Erickson et al. (1979)}$$

$$SCET \text{ factorization}$$

$$W. \text{ Waalewijn (2012)} \qquad D. \text{ Krohn et al. (2013)}$$

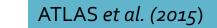
#### Significance: different flavor jets in HIC

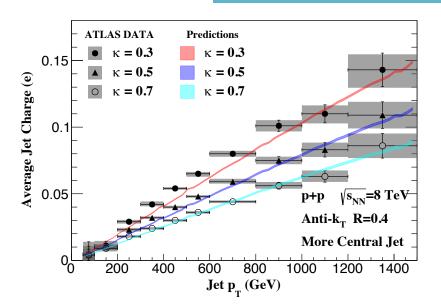


## Phenomenological results in proton collisions

- Calculation of the jet maching coefficient & jet function
- It is important that it can be expressed as an integral over splitting kernels. In medium only numerical grids possible

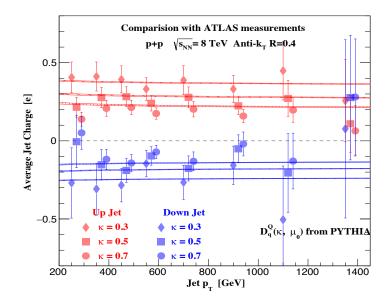
#### **Phenomenology**





$$\begin{split} \mathcal{J}_{qq}^{(1)}(E,R,x,\mu) &= \\ \frac{C_F \alpha_s}{2\pi} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \int \frac{dl_{\perp}^2}{l_{\perp}^2} \left(\frac{\mu^2}{l_{\perp}^2}\right)^{\epsilon} \frac{1+x^2-\epsilon(1-x)^2}{1-x} \\ J_q(E,R,\mu) &= \int_0^1 dz z \left[ \mathcal{J}_{qq}(E,R,z,\mu) + \mathcal{J}_{qg}(E,R,z,\mu) \right] \\ \text{H. Li et al. (2019)} \end{split}$$

$$\mu \frac{d}{d \mu} \tilde{D}_q^Q(\kappa, \mu) = \frac{\alpha_s(\mu)}{\pi} \tilde{P}_{qq}(\kappa) \tilde{D}_q^Q(\kappa, \mu)$$



#### **Derivation in heavy ion collisions**

- Jet matching coefficient in matter
- Note that the virtual correction does not give a contribution. All contained in the LO result

$$\begin{split} \mathcal{J}_{qq}^{\mathrm{med}}(E,R,x,\mu) &= \\ & \frac{\alpha_s(\mu)}{2\pi^2} \left[ -\delta(1-x) \int_0^1 dz \int_0^\mu \frac{d^2 \mathbf{k}_\perp}{\mathbf{k}_\perp^2} P_{q \to qg}^{\mathrm{med}}\left(z,\mathbf{k}_\perp\right) \right. \\ & \left. + \int_0^{2Ex(1-x)\tan R/2} \frac{d^2 \mathbf{k}_\perp}{\mathbf{k}_\perp^2} P_{q \to qg}^{\mathrm{med}}\left(x,\mathbf{k}_\perp\right) \right] \\ & = \frac{\alpha_s(\mu)}{2\pi^2} \int_0^{2Ex(1-x)\tan R/2} \frac{d^2 \mathbf{k}_\perp}{\mathbf{k}_\perp^2} P_{q \to qg}^{\mathrm{med}}\left(x,\mathbf{k}_\perp\right) \end{split}$$

$$\begin{split} J_q^{\text{med}}(E,R,\mu) &= \int_0^1 dx \; x \bigg( \mathcal{J}_{qq}^{\text{med}}(E,R,x,\mu) + \mathcal{J}_{qg}^{\text{med}}(E,R,x,\mu) \bigg) \\ &= \frac{\alpha_s(\mu)}{2\pi^2} \int_0^1 dx \int_0^{2Ex(1-x)\tan R/2} \frac{d^2 \mathbf{k}_\perp}{\mathbf{k}_\perp^2} \bigg( x P_{q \to qg}^{\text{med,real}}\left(x,\mathbf{k}_\perp\right) + x P_{q \to gq}^{\text{med,real}}\left(x,\mathbf{k}_\perp\right) \bigg) \\ &= \frac{\alpha_s(\mu)}{2\pi^2} \int_0^1 dx \int_0^{2Ex(1-x)\tan R/2} \frac{d^2 \mathbf{k}_\perp}{\mathbf{k}_\perp^2} P_{q \to qg}^{\text{med,real}}\left(x,\mathbf{k}_\perp\right), \end{split}$$

 The inmedium jet function

- Medium evolution of the fragmentation function
- Boundary condition obtained from PYTHIA

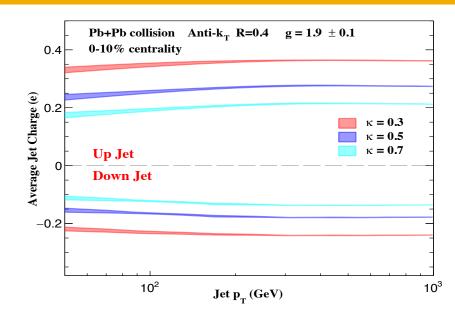
$$\begin{split} \frac{d}{d\ln\mu} \tilde{D}_q^{Q,\text{full}}(\kappa,\mu) &= \\ \frac{\alpha_s(\mu)}{\pi} \left( \tilde{P}_{qq}(\kappa) + \tilde{P}_{qq}^{\text{med}}(\kappa,\mu) \right) \tilde{D}_q^{Q,\text{full}}(\kappa,\mu) \end{split}$$

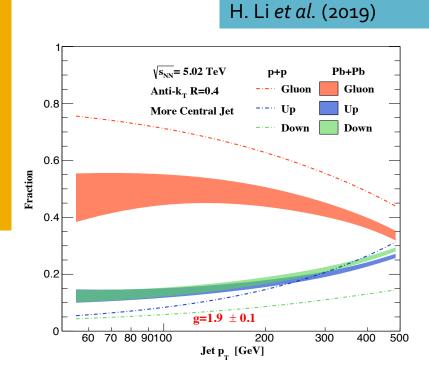
H. Li *et al.* (2019)

## Phenomenological predictions for heavy ion collisions

#### The effects that are important

- Isospin, many more down quarks
- Energy loss effects, quark jets lose less energy than gluon jets (C<sub>F</sub> vs C<sub>A</sub>)
- Medium induced splitting effects on the jet functions ands the fragmentation function evolution



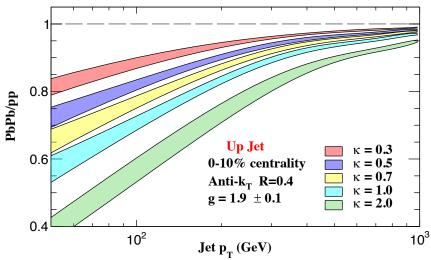


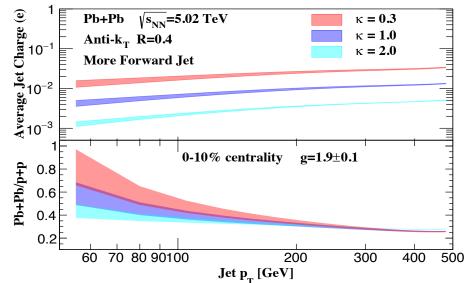
#### Individual flavor jets can still be separated

## Phenomenological predictions for heavy ion collisions

H. Li et al. (2019)

- At very large transverse momenta isospin effects dominate.
- At lower transverse momenta p<sub>T</sub><200 GeV we are beginning to see the effects of inmedium parton showers and different evolution

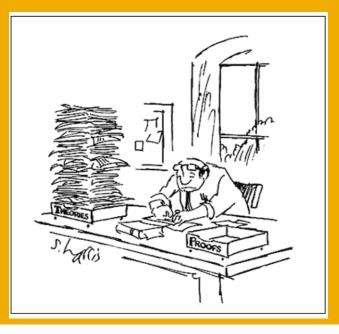




#### Proposed new measurement – the charge of individual flavor jets

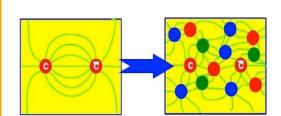
- Isolate the medium induced contribution to jet functions and fragmentation functions evolution.
- Mellin moments of in-medium splittings

## NOCD, Leading power factorization & E-loss

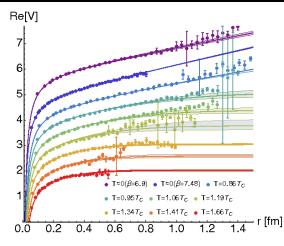


### Quarkonia in the QGP

- Quarkonia (e.g. J/ψ, Y), bound states of the heaviest elementary particles, long considered standard candle to characterize QGP properties
- Most sensitive to the spacetime temperature profile



Matsui *et al. (*1986)



Rothkopf et al. (2016)

$$\left[-\frac{1}{2\mu_{\rm red}}\frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{2\mu_{\rm red}r^2} + V(r)\right]rR_{nl}(r) = (E_{nl} - 2m_Q)rR_{nl}(r)$$

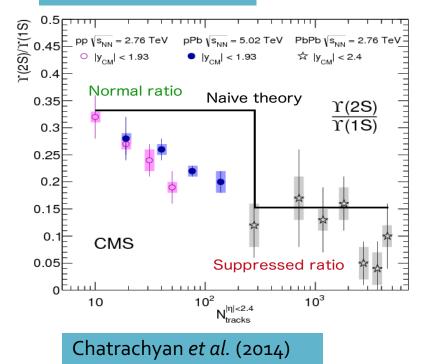
 $\psi(\mathbf{r}) = Y_{l}^{m}(\hat{r})R_{nl}(r)$   $\begin{array}{c} T & 1/\langle r \rangle \\ 450 \text{ MeV} \\ \psi(\mathbf{r}) = Y_{l}^{m}(\hat{r})R_{nl}(r) \\ 450 \text{ MeV} \\ \chi_{b}(1P) \\ 1/\psi(1S) \\ \chi_{c}(1P) \\ \chi_{c}(1P) \end{array}$ 

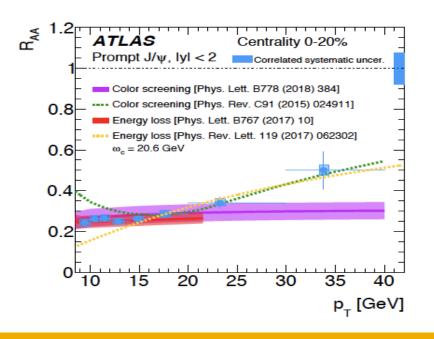
l n	$E_{nl}$ (GeV)	$\sqrt{\langle r^2 \rangle} \; (\mathrm{GeV}^{-1}) \; k^2$	$^{2}$ (GeV <sup>2</sup> )	Meson
0 1	0.700	2.24	0.30	$J/\psi$
$0 \ 2$	0.086	5.39	0.05	$\psi(2S)$
1 1	0.268	3.50	0.20	$\chi_c$
0 1	1.122	1.23	0.99	$\Upsilon(1S)$
$0 \ 2$	0.578	2.60	0.22	$\Upsilon(2S)$
$0 \ 3$	0.214	3.89	0.10	$\Upsilon(3S)$
$1 \ 1$	0.710	2.07	0.58	$\chi_b(1P)$
$1 \ 2$	0.325	3.31	0.23	$\chi_b(2P)$
$1 \ 3$	0.051	5.57	0.08	$\chi_b(3P)$

### **Challenges and hypothesae**

- Suppression puzzle similar dissociation behavior observed in small system, p+A and even in p+p (where QGP is not expected)
- Co-mover dissociation model, energy loss model need cross check and microscopic explanation

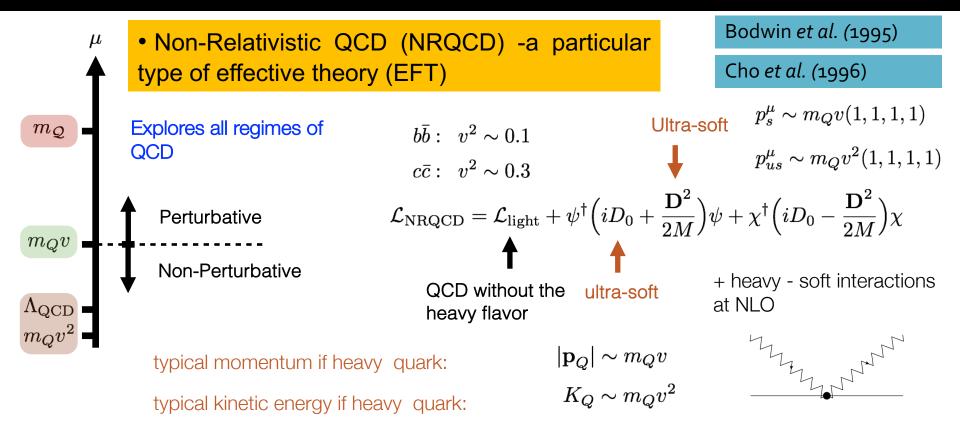
#### E. Ferreiro (2014)





 EFT - capture the interactions without explicitly specifying their nature

## Production of quarkonia at intermediate and high $p_T$



• NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long distance matrix elements (fit to data, scaling relations)

$$d\sigma(a+b\to Q+X) = \sum_{n} d\sigma(a+b\to Q\overline{Q}(n)+X) \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

#### **NRQCD** examples

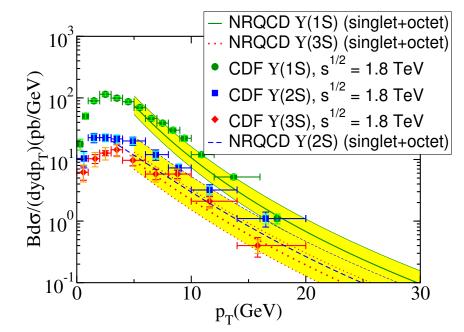
• One has to be careful, the simple power counting approximately manifest in the LDMEs can be affected by the partonic cross section – a large number of singlet and octet; S wave and P wave terms enter

 $d\sigma(J/\psi) = d\sigma(Q\bar{Q}([^{3}S_{1}]_{1}))\langle \mathcal{O}(Q\bar{Q}([^{3}S_{1}]_{1}) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^{1}S_{0}]_{8}))\langle \mathcal{O}(Q\bar{Q}([^{1}S_{0}]_{8}) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^{3}S_{1}]_{8}))\langle \mathcal{O}(Q\bar{Q}([^{3}S_{1}]_{8}) \to J/\psi) \rangle + d\sigma(Q\bar{Q}([^{3}P_{0}]_{8}))\langle \mathcal{O}(Q\bar{Q}([^{3}P_{0}]_{8}) \to J/\psi) \rangle$ 

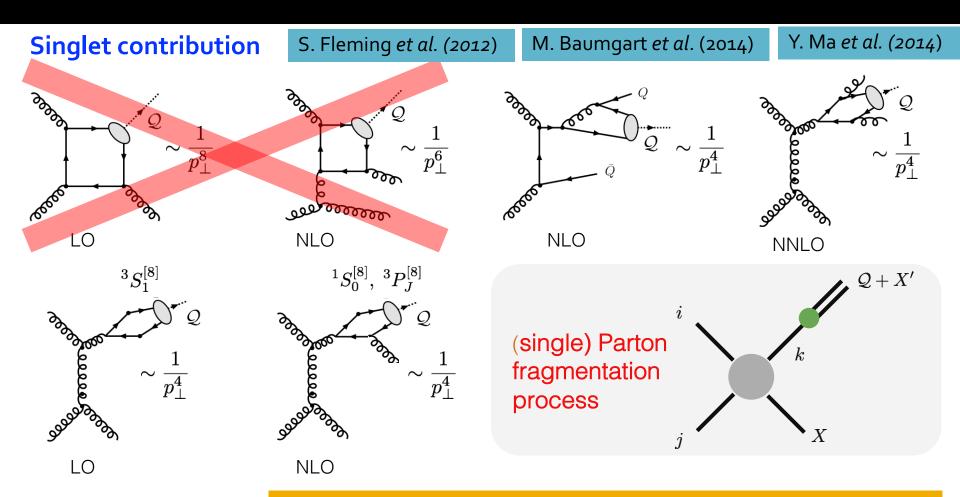
 $+ d\sigma(Q\bar{Q}([^{3}P_{1}]_{8}))\langle \mathcal{O}(Q\bar{Q}([^{3}P_{1}]_{8}) \rightarrow J/\psi)\rangle + d\sigma(Q\bar{Q}([^{3}P_{2}]_{8}))\langle \mathcal{O}(Q\bar{Q}([^{3}P_{2}]_{8}) \rightarrow J/\psi)\rangle + \cdots$ 

The situation is similar for bottomonia
Excited states have their own expansion

The question is – is there a simplification at high  $p_T$  where the  $p_T$  dependence of the short distance cross section dominates



### Leading power factorization



**Octet contribution** 

Only a subset of contributions survive, now interpretable as parton fragmentation in quarkonia

#### LP example and applicability

$$\frac{d\sigma_{h}}{dp_{\perp}}(p_{\perp}) = \sum_{i} \int_{z}^{1} \frac{dx}{x} \frac{d\sigma_{i}}{dp_{\perp}} \left(\frac{p_{\perp}}{x}, \mu\right) D_{i/h}(x, \mu) + \mathcal{O}\left(\frac{m_{h}^{2}}{p_{\perp}^{2}}\right)$$

$$p_{T} \gg m_{Q}$$

$$\ln\left(\frac{\mu}{p_{T}}\right) - \ln\left(\frac{\mu}{2m_{Q}}\right) d_{i/n}(x, \mu) \langle \mathcal{O}_{n}^{h} \rangle$$

$$b_{\parallel} = b_{\parallel}$$

$$\frac{d}{d\mu} D_{i/h}(z, \mu) = \sum_{i} \int_{z}^{1} \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$
Resummation of  $\ln(p_{T}/m_{h})$ 

$$\frac{d}{d\mu} \frac{d}{d\mu} D_{i/h}(z, \mu) = \sum_{i} \int_{z}^{1} \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

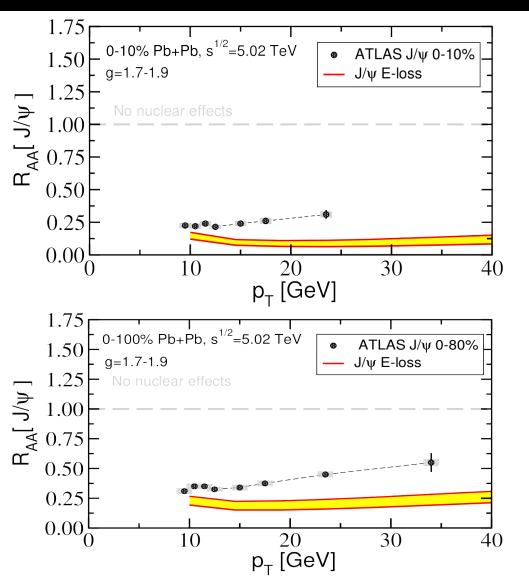
$$\frac{d}{d\mu} \frac{d}{d\mu} \frac$$

### Comparison of energy loss phenomenology to data

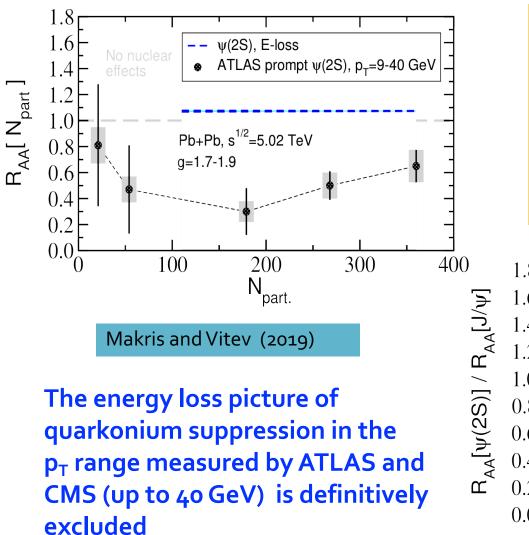
- Suppression of J/ψ overestimated by factor of 2 to 3. Included χ<sub>c</sub> and ψ(2S) feeddown.
- Persists over centralities. Somewhat different p<sub>T</sub> dependence
- Differences are significan

$$R_{AA}^{\min. \text{ bias}}(p_T) = \frac{\sum_i R_{AA}(\langle b_i \rangle) W_i}{\sum_i W_i}$$

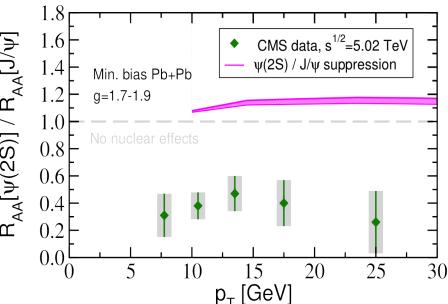
$$W_i = \int_{b_{i \min}}^{b_{i \max}} N_{\text{coll.}}(b) \, \pi \, b \, db$$



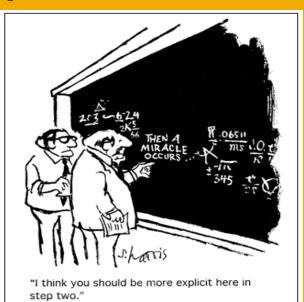
### Double suppression ratio $\psi(2S) / J/\psi$



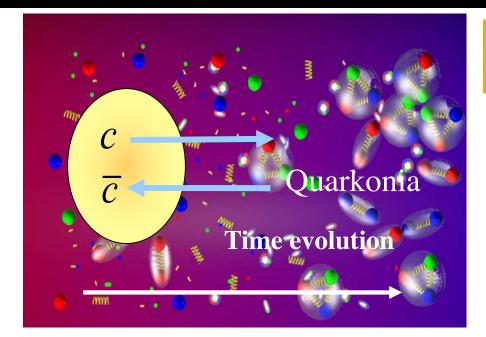
 In the double suppression ratio R<sub>AA</sub>(ψ(2S))/R<sub>AA</sub>(J/ψ) the discrepancy is not simply in magnitude. There is a discrepancy in the sign of prediction



### NRQCD with Glauber Gluons & phenomenology



### NRQCD in a background medium



 Take a closer look at the NRQCD Lagrangian below

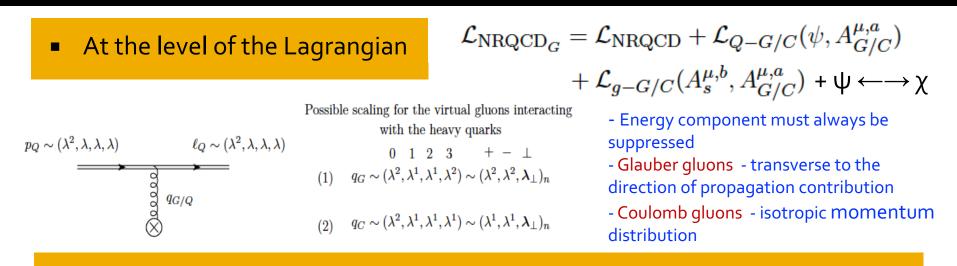
 $\label{eq:scales} \begin{array}{ll} \mbox{Scales in the problem} \\ p_s^\mu \sim m_Q v(1,1,1,1) & \mbox{soft} \sim \lambda \\ p_{us}^\mu \sim m_Q v^2(1,1,1,1) & \mbox{ultrasoft} \sim \lambda^2 \end{array}$ 

 Ultrasoft gluons included in covariant derivatives

- Soft gluons are included explicitly
- Double soft gluon emission
- Heavy quark-antiquark potential
- (can also be interaction with soft particles)

$$\begin{split} \mathcal{L} &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{p} \left| p^{\mu} A_{p}^{\nu} - p^{\nu} A_{p}^{\mu} \right|^{2} + \sum_{\mathbf{p}} \psi_{\mathbf{p}}^{\dagger} \left\{ i D^{0} - \frac{(\mathbf{p} - i \mathbf{D})^{2}}{2m} \right\} \psi_{\mathbf{p}} \\ &- 4\pi \alpha_{s} \sum_{q,q'\mathbf{p},\mathbf{p}'} \left\{ \frac{1}{q^{0}} \psi_{\mathbf{p}'}^{\dagger} \left[ A_{q'}^{0}, A_{q}^{0} \right] \psi_{\mathbf{p}} \right. \\ &+ \frac{g^{\nu 0} \left( q' - p + p' \right)^{\mu} - g^{\mu 0} \left( q - p + p' \right)^{\nu} + g^{\mu \nu} \left( q - q' \right)^{0}}{\left( \mathbf{p}' - \mathbf{p} \right)^{2}} \psi_{\mathbf{p}'}^{\dagger} \left[ A_{q'}^{\nu}, A_{q}^{\mu} \right] \psi_{\mathbf{p}} \right\} \\ &+ \psi \leftrightarrow \chi, \ T \leftrightarrow \bar{T} \\ &+ \sum_{\mathbf{p},\mathbf{q}} \frac{4\pi \alpha_{s}}{\left( \mathbf{p} - \mathbf{q} \right)^{2}} \psi_{\mathbf{q}}^{\dagger} T^{A} \psi_{\mathbf{p}} \chi_{-\mathbf{q}}^{\dagger} \bar{T}^{A} \chi_{-\mathbf{p}} + \dots \end{split}$$

#### Allowed interactions in the medium



 Calculated the leading power and next to leading power contributions 3 different ways

Background field method	Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting
Hybrid method	From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules
Matching method	Full QCD diagrams describing the forward scattering of incoming heavy quark and a light quark or a gluon. We also derive the tree level expressions of the effective fields in terms of the QCD ingredients

## Example of the background field method

 Perform the label momentum representation and field substitution (u.s. -> u.s. + Glauber)

 $\mathbf{E} = \partial_t (\mathbf{A}_U + \mathbf{A}_G) + (\partial + i\boldsymbol{\mathcal{P}})(A_U^0 + A_G^0) + gT^c f^{cba} (A_U^0 + A_G^0)^b (\mathbf{A}_U + \mathbf{A}_G)^a$ 

$$\psi(x) \to \sum_{\mathbf{p}} \psi_{\mathbf{p}}(x) ,$$
  
 $iD_{\mu} \to \mathcal{P}_{\mu} + i\partial_{\mu} - g(A_U^{\mu} + A_{G/C}^{\mu}) .$ 

Example for a collinear source (note results depend on the type of source)

Substitute, expand and collect terms up to order  $\lambda^{3}$ 

 Results: depend on the type of the source of scattering in the medium

 $iD_t = \underbrace{i\partial_t - gA_U^0 - gA_G^0}_{\sim \lambda^2},$ 

 $=\underbrace{i {\cal P}_{\perp} A^0_G}_{\sim \lambda^3} + {\cal O}(\lambda^4) \; ,$ 

 $= -\underbrace{(i \mathcal{P}_{\perp} \times \mathbf{n}) \ A^{\mathbf{n}}_{G}}_{\sim \lambda^{3}} + \mathcal{O}(\lambda^{4}) \ .$ 

 $i\mathbf{D} = \underbrace{\mathcal{P}}_{\sim \lambda} - (\underbrace{i\partial + g\mathbf{A}_U + g\mathbf{n}A_G^{\mathbf{n}}}_{\sim \lambda^2}) + \mathcal{O}(\lambda^3) ,$ 

Leading medium corrections Sub-leading medium corrections

 $\mathbf{B} = -(\partial + i\boldsymbol{\mathcal{P}}) \times (\mathbf{A}_U + \mathbf{A}_G) + \frac{g}{2}T^c f^{cba} (\mathbf{A}_U + \mathbf{A}_G)^b (\mathbf{A}_U + \mathbf{A}_G)^a$ 

$$\begin{split} \mathcal{L}_{Q-G/C}^{(0)}(\psi, A_{G/C}^{\mu, a}) &= \sum_{\mathbf{p}, \mathbf{q}_{T}} \psi_{\mathbf{p}+\mathbf{q}_{T}}^{\dagger} \left( -gA_{G/C}^{0} \right) \psi_{\mathbf{p}} \ (collinear/static/soft). \\ \mathcal{L}_{Q-G}^{(1)}(\psi, A_{G}^{\mu, a}) &= g \sum_{\mathbf{p}, \mathbf{q}_{T}} \psi_{\mathbf{p}+\mathbf{q}_{T}}^{\dagger} \left( \frac{2A_{G}^{\mathbf{n}}(\mathbf{n} \cdot \boldsymbol{\mathcal{P}}) - i \left[ (\boldsymbol{\mathcal{P}}_{\perp} \times \mathbf{n}) A_{G}^{\mathbf{n}} \right] \cdot \boldsymbol{\sigma}}{2m} \right) \psi_{\mathbf{p}} \ (collinear) \\ \mathcal{L}_{Q-C}^{(1)}(\psi, A_{C}^{\mu, a}) &= 0 \ (static) \\ \mathcal{L}_{Q-C}^{(1)}(\psi, A_{C}^{\mu, a}) &= g \sum_{\mathbf{p}, \mathbf{q}_{T}} \psi_{\mathbf{p}+\mathbf{q}_{T}}^{\dagger} \left( \frac{2\mathbf{A}_{G}^{\mathbf{n}}(\mathbf{n} \cdot \boldsymbol{\mathcal{P}}) - i \left[ (\boldsymbol{\mathcal{P}}_{\perp} \times \mathbf{n}) A_{G}^{\mathbf{n}} \right] \cdot \boldsymbol{\sigma}}{2m} \right) \psi_{\mathbf{p}} \ (soft) \end{split}$$

# The QCD forward scattering diagram expansion

 Looking at t-channel scattering we can also extract the form of the Glauber/Coulomb fields in terms of QCD ingredients (and recover Lagrangian)

**Glauber field for collinear source** 

$$A_G^{\mu,a} = \frac{n^\mu}{\mathbf{q}_T^2} \sum_{\ell} \bar{\xi}_{n,\ell-\mathbf{q}_T} \frac{\not{n}}{2} (gT^a) \xi_{n,\ell}$$

**Coulomb field for soft source** 

$$A_C^{\mu,a} \equiv \frac{1}{\mathbf{q}^2} \sum_{\ell} \bar{\phi}_{\ell-\mathbf{q}} \gamma^{\mu} (gT^A) \phi_{\ell}$$

 $t_{g-coll.} = \frac{p'}{p'_n} \xrightarrow{p}_n + \underbrace{p'_n}_{p_n} + \underbrace{p'_n}_{$ 

Glauber field for collinear source

$$A_{G}^{\mu,a} = \frac{i}{2}gf^{abc}\frac{n^{\mu}}{\mathbf{q}_{T}^{2}}\sum_{\ell}\left[\bar{n}\cdot\mathcal{P}\left(\mathbf{B}_{n\perp,\ell-\mathbf{q}_{T}}^{b(0)}\cdot\mathbf{B}_{n\perp,\ell}^{c(0)}\right)\right]$$

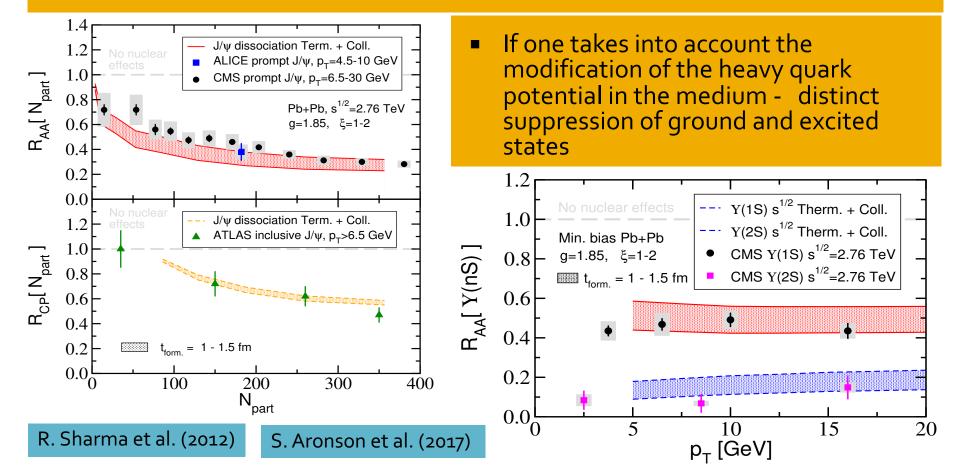
**Coulomb field for soft source** 

Y. Makris et al. (2019)  $A_{C}^{\mu,a} = f^{abc} \frac{ig}{2 \mathbf{q}^{2}} \sum_{\ell} \left\{ \left[ \mathcal{P}^{\mu} \left( \mathbf{B}_{s,\ell-\mathbf{q}}^{b(0)} \cdot \mathbf{B}_{s,\ell}^{c(0)} \right) \right] - 2(\mathbf{B}_{s,\ell}^{c(0)} \cdot \left[ \boldsymbol{\mathcal{P}} \right) B_{s,\ell-\mathbf{q}}^{\mu,b(0)} \right] - 2(\mathbf{B}_{s,\ell-\mathbf{q}}^{b(0)} \cdot \left[ \boldsymbol{\mathcal{P}} \right) B_{s,\ell-\mathbf{q}}^{b(0)} \cdot \left[ \boldsymbol{\mathcal{P}} \right) B_{s,\ell-\mathbf{q}}^{b(0)} \cdot \left[ \boldsymbol{\mathcal{P}} \right] B_{s,\ell-\mathbf{q}}^{\mu,c(0)} \right] \right\}$ 

- Note that for the gluon the last 2 diagrams are necessary for gauge invariance but the first diagram the leading forward scattering contribution
- In the medium the momentum exchange can get dressed ~ Debye screening

### Possible phenomenology applications

 Phenomenology built so far is cobnnected the leading term – collisional dissociation, thermal effects put in quarkonium wavefunctions. Also there is the approximation of averaging over all final color states

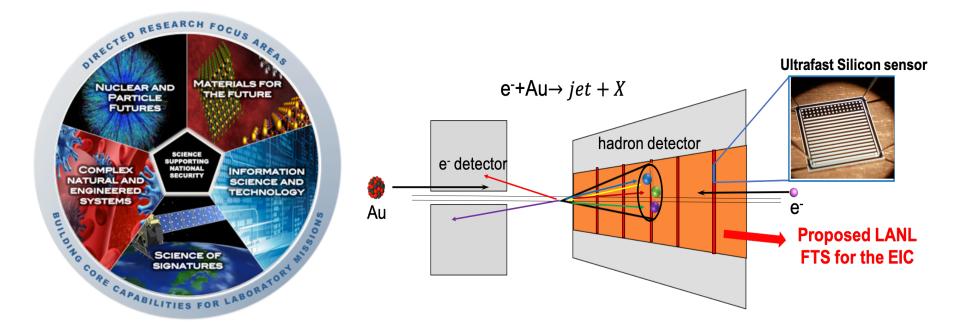


### Conclusions

- Effective theories of QCD have enabled important conceptual and technical breakthroughs in our understanding of strong interactions and very significant improvement in the accuracy of the theoretical predictions
- An effective theory for jet propagation in matter SCET<sub>G</sub> was constructed (collinear and Glauber sectors). Derived all medium-induced parton splittings now to any order in opacity. Developed a new code for plitting kernel grids to second order in opacity
- Performed the first calculation of inclusive heavy jet production (c-jets, b-jets) in heavy ion reactions using the semi-inclusive jet function approach and presented a framework/evaluation of the jet charge in reactions ith nuclei
- In the the leading power factorization (high p<sub>T</sub>) limit of NRQCD we investigated energy loss phenomenology and showed that it severely overpredicts the J/ψ modification and gives the wrong hierarchy of ground/excited suppression
- Motivated by this we constructed an effective theory of quarkonia in matter
   NROCD<sub>G</sub> Derived the Feynman rules (3 different ways) to leading and subleading power for different sources of interactions in the medium. We showed the connection to existing quarkonium dissociation phenomenology

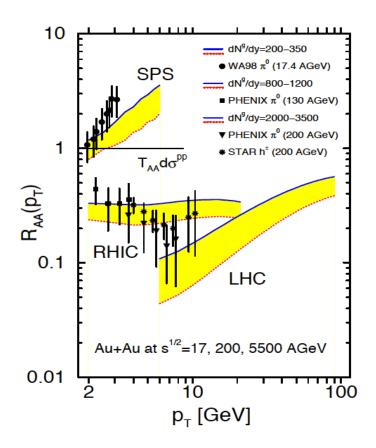
### Conclusions

- LANL just made a very large investment in EIC science - \$1.5M over 3 years
- Comes in the form of a LDRD project.
   PI. I. Vitev, Co-PI Xuan Li
- Develop a prototype forward silicon tracker. Heavy flavor physics and jets
- Develop associated theory



### Improving upon traditional E-loss

 While still LO, it predicted in 2002, 2006 – the R<sub>AA</sub> at high p<sub>T</sub> for both RHIC and LHC



Include the quenched parton and the radiative gluon fragmentation

- Difficult to make connection to the standard LO, NLO, ...; LL, NLL ... pQCD approach (higher orders and resummation)
- There is considerable model dependence and it is difficult to systematically improve this approach

I. Vitev et al. (2002)

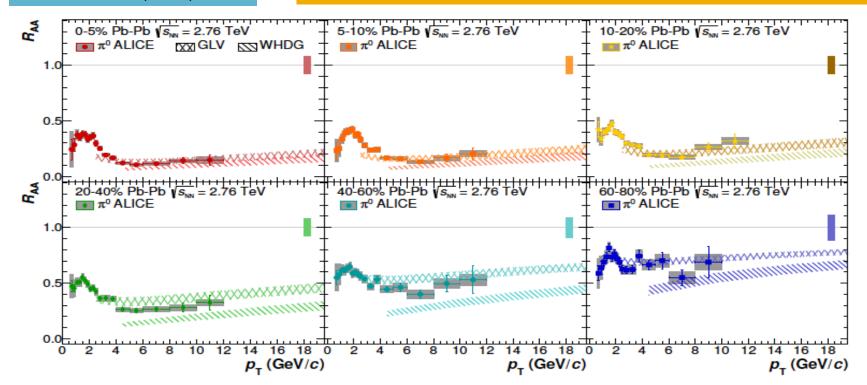
### Reminder – traditional energy loss phenomenology

$$\int_0^1 d\epsilon \ P(\epsilon) = 1 \ , \quad \int_0^1 d\epsilon \ \epsilon \ P(\epsilon) = \left\langle \frac{\Delta E}{E} \right\rangle \qquad D_c^{\text{quench}}(z) = \int_0^{1-z} \ d\epsilon \ \frac{P_c(\epsilon)}{(1-\epsilon)} D_c\left(\frac{z}{1-\epsilon}\right)$$

M. Gyulassy et al. (2002)

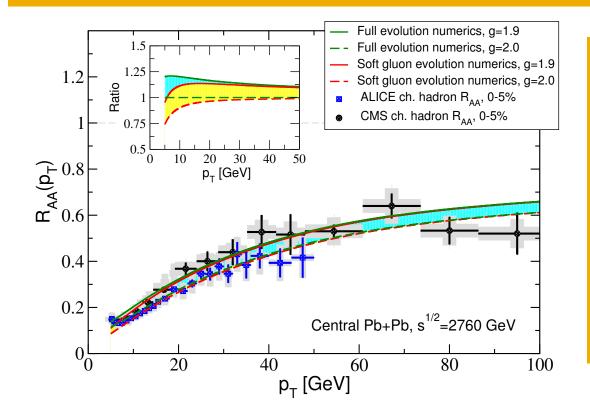
Successfully predicted the energy and transverse momentum dependence of R<sub>AA</sub>

ALICE Collab. (2014)



# Numerical results: full-x vs small-x evolution

 Implement the fully numerical solution of the DGLAP evolution equations: the full splitting kernels and the soft gluon limit (small x)



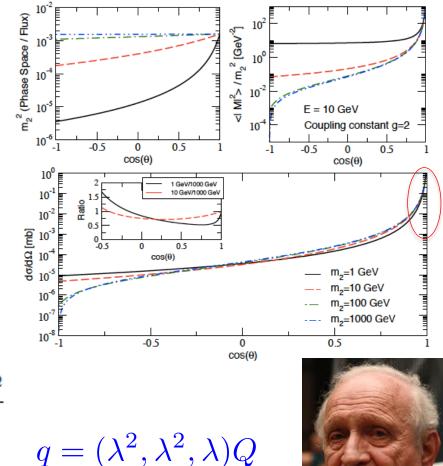
- The coupling between the jet and the medium can be constrained to the same accuracy - 5%
- Full evolution woks slightly better at low virtualities

### The jet scattering kinematics

- Kinematics and channels
- t jet broadening and energy loss
- s-isotropisation
- u backward hard scattering
- Fully dynamic medium recoil, cross section reduction (5% -15%). Completely dominated by forward scattering

$$\frac{d\sigma}{d\Omega} \to \frac{d\sigma}{d^2 \mathbf{q}_\perp} = \frac{C_2(R)C_2(T)}{d_A} \frac{|v(\mathbf{q}_\perp; E, m_1, m_2)|^2}{(2\pi)^2}$$

Galuber gluon / Glauber mode 



A. Idilbi et al. (2008)



### In-medium parton splittings and properties

Direct sum

 $\frac{dN(tot.)}{dxd^2k_{\perp}} = \frac{dN(vac.)}{dxd^2k_{\perp}} + \frac{dN(med.)}{dxd^2k_{\perp}}$ 

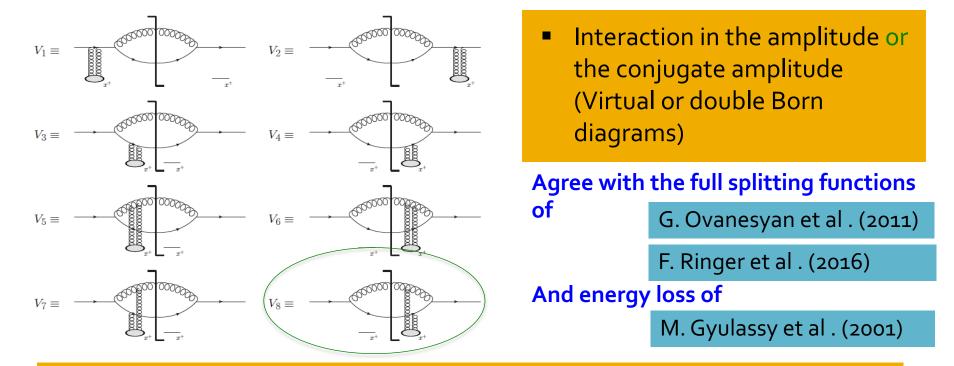
- Derived using SCET<sub>G</sub>
- Factorize form the hard part
- Gauge-invariant
- Depend on the properties of the medium

G. Ovanesyan et al. (2012)

$$\begin{split} \left(\frac{dN}{dxd^{2}\boldsymbol{k}_{\perp}}\right)_{q \to qg} &= \frac{\alpha_{s}}{2\pi^{2}}C_{F}\frac{1+(1-x)^{2}}{x}\int\frac{d\Delta z}{\lambda_{g}(z)}\int d^{2}\mathbf{q}_{\perp}\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\mathrm{medium}}}{d^{2}\mathbf{q}_{\perp}}\left[-\left(\frac{A_{\perp}}{A_{\perp}^{2}}\right)^{2}+\frac{B_{\perp}}{B_{\perp}^{2}}\cdot\left(\frac{B_{\perp}}{B_{\perp}^{2}}-\frac{C_{\perp}}{C_{\perp}^{2}}\right)\right.\\ &\times\left(1-\cos[(\Omega_{1}-\Omega_{2})\Delta z]\right)+\frac{C_{\perp}}{C_{\perp}^{2}}\cdot\left(2\frac{C_{\perp}}{C_{\perp}^{2}}-\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{B_{\perp}}{B_{\perp}^{2}}\right)\left(1-\cos[(\Omega_{1}-\Omega_{3})\Delta z]\right)\\ &+\frac{B_{\perp}}{B_{\perp}^{2}}\cdot\frac{C_{\perp}}{C_{\perp}^{2}}\left(1-\cos[(\Omega_{2}-\Omega_{3})\Delta z]\right)+\frac{A_{\perp}}{A_{\perp}^{2}}\cdot\left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{D_{\perp}}{D_{\perp}^{2}}\right)\cos[\Omega_{4}\Delta z]\\ &+\frac{A_{\perp}}{A_{\perp}^{2}}\cdot\frac{D_{\perp}}{D_{\perp}^{2}}\cos[\Omega_{5}\Delta z]+\frac{1}{N_{c}^{2}}\frac{B_{\perp}}{B_{\perp}^{2}}\cdot\left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{B_{\perp}}{B_{\perp}^{2}}\right)\left(1-\cos[(\Omega_{1}-\Omega_{2})\Delta z]\right)\right]. \end{split}$$

$$\begin{split} N.B. \ x &\rightarrow 1-x \quad A, \dots D, \Omega_{1} \dots \Omega_{5} - functions(x, k_{\perp}, q_{\perp}) \\ \left(\frac{dN}{dxd^{2}k_{\perp}}\right)_{\left\{\begin{array}{l}g \rightarrow q\bar{q}\\g \rightarrow gg\end{array}\right\}} = \begin{cases} \frac{\alpha_{s}}{2\pi^{2}}T_{R}\left(x^{2}+(1-x)^{2}\right) \\ \frac{\alpha_{s}}{2\pi^{2}}2C_{A}\left(\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right) \end{cases} \\ \int d\Delta z \left\{\begin{array}{l}\frac{1}{\lambda_{q}(z)} \\ \frac{1}{\lambda_{g}(z)}\end{array}\right\} \int d^{2}\mathbf{q}_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^{2}\mathbf{q}_{\perp}} \\ \times \left[2\frac{B_{\perp}}{B_{\perp}^{2}} \cdot \left(\frac{B_{\perp}}{B_{\perp}^{2}}-\frac{A_{\perp}}{A_{\perp}^{2}}\right)\left(1-\cos\left[(\Omega_{1}-\Omega_{2})\Delta z\right]\right)+2\frac{C_{\perp}}{C_{\perp}^{2}} \cdot \left(\frac{C_{\perp}}{C_{\perp}^{2}}-\frac{A_{\perp}}{A_{\perp}^{2}}\right)\left(1-\cos\left[(\Omega_{1}-\Omega_{3})\Delta z\right]\right) \\ + \left\{\frac{1}{N_{c}^{2}-1}\right\} \left(2\left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{B_{\perp}}{B_{\perp}^{2}}\right) \cdot \left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{C_{\perp}}{C_{\perp}^{2}}\right)+2\frac{B_{\perp}}{B_{\perp}^{2}} \cdot \left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{C_{\perp}}{C_{\perp}^{2}}\right)\cos\left[(\Omega_{1}-\Omega_{3})\Delta z\right] \\ +2\frac{C_{\perp}}{C_{\perp}^{2}} \cdot \left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{B_{\perp}}{B_{\perp}^{2}}\right)\cos\left[(\Omega_{1}-\Omega_{3})\Delta z\right]+2\frac{C_{\perp}}{C_{\perp}^{2}} \cdot \frac{B_{\perp}}{B_{\perp}^{2}}\cos\left[(\Omega_{2}-\Omega_{3})\Delta z\right] \\ -2\frac{A_{\perp}}{A_{\perp}^{2}} \cdot \left(\frac{A_{\perp}}{A_{\perp}^{2}}-\frac{D_{\perp}}{D_{\perp}^{2}}\right)\cos\left[\Omega_{4}\Delta z\right]-2\frac{A_{\perp}}{A_{\perp}^{2}} \cdot \frac{D_{\perp}}{D_{\perp}^{2}}\cos\left[\Omega_{5}\Delta z\right]\right) \right]. \end{split}$$

### Opacity expansion building blocks –virtual terms

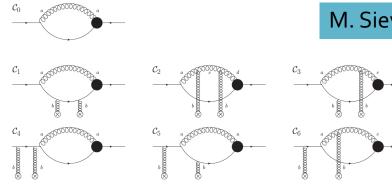


A more interesting diagram- Double born can contribute to virtuality changes

$$V_8 = \left[\frac{N_c}{2C_F} e^{i[\Delta E^-(\underline{k}-x\underline{p})-\Delta E^-(\underline{k}-x\underline{p}-\underline{q})]z^+}\right] \psi(x,\underline{k}-x\underline{p}) \left[0-e^{-i\Delta E^-(\underline{k}-x\underline{p})x_0^+}\right] \\ \times \left[e^{+i\Delta E^-(\underline{k}-x\underline{p}-\underline{q})z^+}-e^{+i\Delta E^-(\underline{k}-x\underline{p}-\underline{q})x_0^+}\right] \psi^*(x,\underline{k}-x\underline{p}-\underline{q}).$$

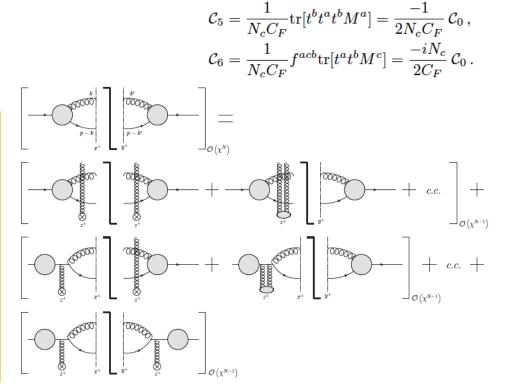
# Parton branching to any order in opacity

Treating color (one complication in QCD).



- Color is not entangled, homogeneous structure and multiplicative factors that can be algebraically treated
- Finally, relative to the splitting vertex we classify the as
- Initial/Initial, Initial/Final, Final/Initial and Final/Final

M. Sievert et al . (2018)



 $\mathcal{C}_1 = \frac{1}{N_c C_F} \operatorname{tr}[t^b t^b t^a M^a] = \mathcal{C}_0 \,,$ 

 $\mathcal{C}_4 = \frac{1}{N_* C_E} \operatorname{tr}[t^a t^b t^b M^a] = \mathcal{C}_0 \,,$ 

 $\mathcal{C}_2 = \frac{1}{N_c C_F} f^{acb} f^{cdb} \operatorname{tr}[t^a M^d] = -\frac{N_c}{C_F} \mathcal{C}_0 \,,$ 

 $\mathcal{C}_3 = \frac{1}{N_c C_F} f^{acb} \text{tr}[t^b t^a M^c] = \frac{i N_c}{2 C_F} \mathcal{C}_0 \,,$ 

# Explicit solution to second order in opacity

 Present the first exact result to this order (including the ability to discuss broad or narrow sources)

$$\begin{split} xp^{+} \frac{dN}{d^{2}k \, dx \, d^{2}p \, dp^{+}} \Big|_{\mathcal{O}(\chi^{2})} &= \frac{C_{F}}{2(2\pi)^{3}(1-x)} \int_{x_{0}^{+}}^{R^{+}} \frac{dz_{2}^{+}}{\lambda^{+}} \int_{x_{0}^{+}}^{z_{2}^{+}} \frac{dz_{1}^{+}}{\lambda^{+}} \int \frac{d^{2}q_{1}}{\sigma_{el}} \frac{d\sigma^{el}}{d^{2}q_{1}} \frac{d\sigma^{el}}{d^{2}q_{2}} \times \left\{ \left( p^{+} \frac{dN_{0}}{d^{2}p \, dp^{+}} \right) \mathcal{N}_{1} \right. \\ &+ \left( p^{+} \frac{dN_{0}}{d^{2}(p-q_{1}) \, dp^{+}} \right) \mathcal{N}_{2} + \left( p^{+} \frac{dN_{0}}{d^{2}(p-q_{2}) \, dp^{+}} \right) \mathcal{N}_{3} + \left( p^{+} \frac{dN_{0}}{d^{2}(p-q_{1}-q_{2}) \, dp^{+}} \right) \mathcal{N}_{4} \right\} \\ \mathcal{N}_{1} = \end{split}$$

$$\begin{aligned} \left|\psi(\underline{k}-x\underline{p})\right|^{2} \left[\frac{(C_{F}+N_{c})^{2}}{C_{F}^{2}} - \frac{N_{c}(C_{F}+N_{c})}{C_{F}^{2}}\cos(\delta z_{1}\Delta E^{-}(\underline{k}-x\underline{p})) + \frac{N_{c}^{2}}{2C_{F}^{2}}\cos(\delta z_{2}\Delta E^{-}(\underline{k}-x\underline{p})) \\ - \frac{N_{c}(2C_{F}+N_{c})}{2C_{F}^{2}}\cos((\delta z_{1}+\delta z_{2})\Delta E^{-}(\underline{k}-x\underline{p}))\right] & +9 \text{ more pages} \end{aligned}$$

 For broad sources and in the soft gluon limit we have checked that the result reduces to the GLV second order in opacity

### **Corrections in QCD medium**

Collisional energy loss evaluated from operator definition. Included in the LO splitting function

$$J_{J_Q/i}^{\mathrm{med},(0)}(z, p_T, \delta p_T^i) = z \delta_{iQ} \left[ \delta \left( 1 - z - \frac{\delta p_T^i}{p_T + \delta p_T^i} \right) - \delta (1 - z) \right]$$

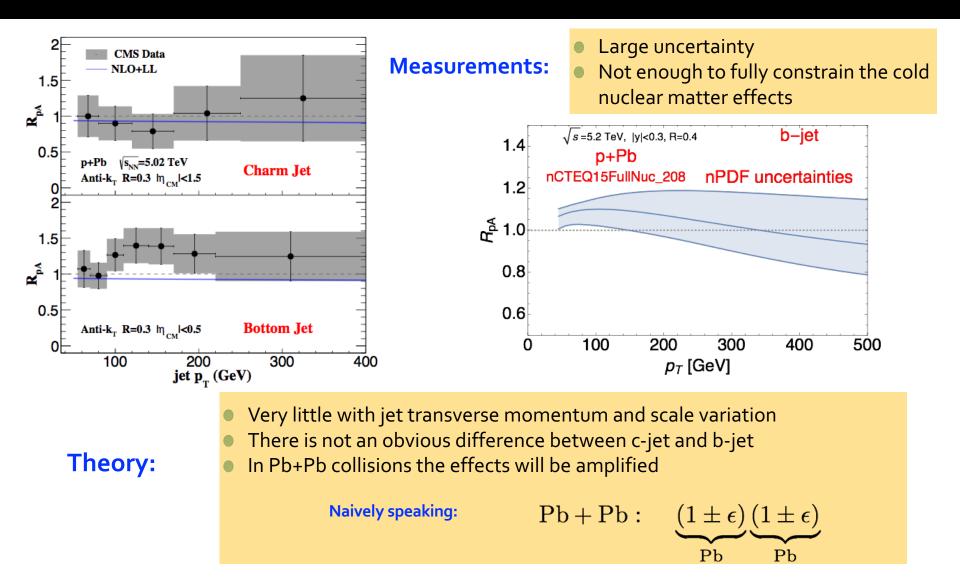
Medium corrections to the NLO jet function are written in terms of integrals over splitting functions. First developed for light jets.

Kang, Ringer, Vitev, 2017

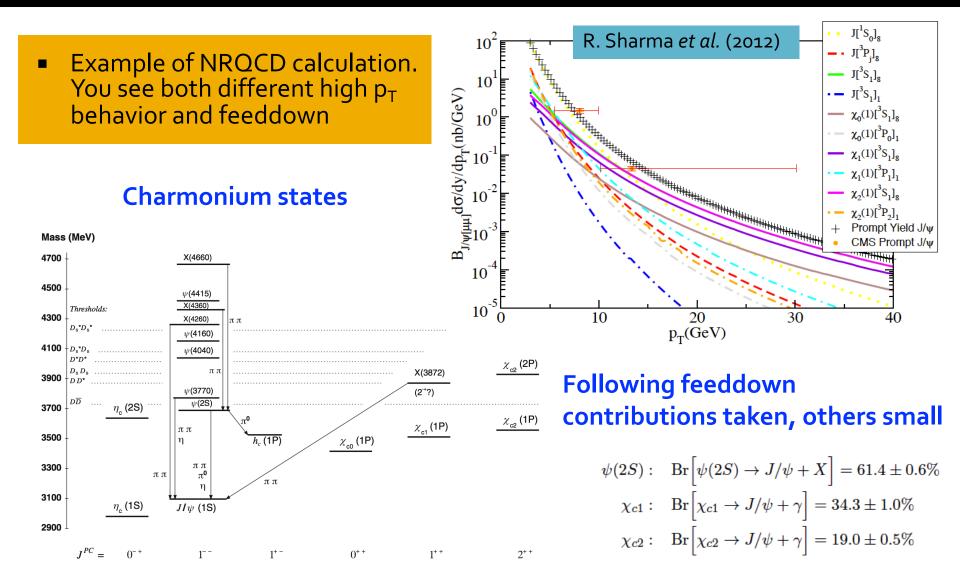
Neufeld, Vitev, Xing, 2014

Full in-medium splitting functions are now evaluated in the hydro medium

## R<sub>AA</sub> in p-A collisions



### **Feeddown is important**



### Energy loss results for quarkonia, constraints



"I'm firmly convinced that behind every great man is a great computer."

### **Energy loss evaluation in** hydrodynamic medium

 Evaluate the splitting functions in the small x limit – corresponds to traditional energy loss phenomenology

Z. Kang *et al.* (2016)

Can make contact with other claims in the literature

**Evaluate the medium-induced emission spectrum.** Construct the probability of energy loss due to multiple gluon emission

$$\int_0^1 d\epsilon P(\epsilon) = 1, \qquad \int_0^1 d\epsilon \epsilon P(\epsilon) = \left\langle \frac{\Delta E}{E} \right\rangle$$

$$\int_0 d\epsilon \epsilon P(\epsilon) = \left\langle \overline{E} \right\rangle$$

Obtain quenched partonic spectra with effective mass m<sub>c</sub> and 2m<sub>c</sub> where necessary

$$\frac{d\sigma_{AB}^{q,g \; Quench}(\mathbf{p})}{dy d^2 \mathbf{p}} = \int_0^1 d\epsilon \; P(\epsilon) \frac{1}{(1-\epsilon)} \frac{d\sigma_{AB}^{q,g}\left(\frac{\mathbf{p}}{1-\epsilon}\right)}{dy d^2 \mathbf{p}}$$

- 10 0.3 [emperature [GeV 5 0.2 [fm] 0 -5 0.1 -10t = 2.0 fm/c0.0 10 -10-5 0 5 [fm] M. Gyulassy et al. (2003) C. Shen *et al.* (2014)
  - Viscous second order Israel-Stewart event-by-event hydrodynamics

### Comparison of energy loss vs dissociation models

 Completely different predictions for ground and excited states' suppression. Dissociation models depend on bunding, energy loss models depend on the flavor of partonic cross sections as steepness of spectra

