Extension of JAM to TMDs

Alexei Prokudin

July 22, 2014
How **partons move** inside of the proton?

Technically such information is encoded into Transverse Momentum Dependent distributions

$$f(x, k_{\perp})$$

These distributions are also referred to as **3D (three-dimensional) distributions**

**TMDs**

- **radiative gluons/sea**
- **sea quarks gluons**
- **valence quarks gluons**
Collinear

$Q^2$ ensures hard scale, pointlike interaction

$P_{hT}$ final hadron transverse momentum can be varied independently

Connection to 3D structure

$\tilde{f}(x, \vec{b}_T) = \int d^2k_\perp e^{-i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$

JAM result on helicity pdfs

$\Delta u(x)$

$\chi$

Jimenez-Delgado et al. (2013)


Ji, Ma, Yuan (2004)

Collins (2011)

AP (2012)
Complementarity of SIDIS, e+e- and Drell-Yan

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them.

- **Semi Inclusive DIS** – convolution of distribution functions and fragmentation functions
  \[ f(x) \otimes D(z) \]
  \[ \ell + P \rightarrow \ell' + h + X \]

- **Drell-Yan** – convolution of distribution functions
  \[ f(x_1) \otimes f(x_2) \]
  \[ P_1 + P_2 \rightarrow \bar{\ell} \ell + X \]

- **e+ e- annihilation** – convolution of fragmentation functions
  \[ D(z_1) \otimes D(z_2) \]
  \[ \bar{\ell} + \ell \rightarrow h_1 + h_2 + X \]

Combining measurements from all above is important.
Global analysis of helicity PDFs

Completed project with
Christopher Lefky
(undergraduate student from Creighton University) who spent summer 2013 at JLab.

Extraction of the pretzelosity distribution from experimental data

Christopher Lefky¹,², * and Alexei Prokudin¹,†

¹Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA
²Creighton University, Omaha NE

(Dated: Monday 3rd February, 2014)

We attempt an extraction of the pretzelosity distribution ($h_{1T}$) from preliminary HERMES, COMPASS and JLAB experimental data on $\sin(3\phi_h - \phi_S)$ asymmetry on proton and deuterium targets. The resulting distributions, albeit big errors, show tendency for up quark pretzelosity to be positive and down quark pretzelosity to be negative. A model relation of pretzelosity distribution and Orbital Angular Momentum of quarks is used to estimate OAM for up and down quarks.

FIG. 2. First moment of the pretzelosity distribution for up (a) and down (b) quarks. The red solid line corresponds to the best fit and the shadowed region corresponds to the error corridor.
What next?
• Collaboration with experimentalists on a TMD database

• Comprehensive framework of TMD extractions

**Time frame**

~2015 database, ~2016 framework
Database
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMPASS</td>
<td>proton and deuteron A1, g1</td>
</tr>
<tr>
<td>EMC</td>
<td>proton A1, g1</td>
</tr>
<tr>
<td>HERMES</td>
<td>proton, deuteron, and 3He A-par, neutron A1</td>
</tr>
<tr>
<td>HERMES2012</td>
<td>proton A2 and g2</td>
</tr>
<tr>
<td>JLab Hall A (E01-012)</td>
<td>3He A-par, A1, g1</td>
</tr>
<tr>
<td>JLab Hall A (E97-103)</td>
<td>3He (and neutron) asymmetries, g1, g2</td>
</tr>
<tr>
<td>JLab Hall A (E99-117)</td>
<td>3He (and neutron) asymmetries, g1, g2</td>
</tr>
<tr>
<td>JLab Hall B (EG11b)</td>
<td>proton and deuteron A1</td>
</tr>
<tr>
<td>JLab Hall C (E01-006 &quot;RSS&quot;)</td>
<td>proton and deuteron A_parallel and A_perp (resonance region)</td>
</tr>
<tr>
<td>SLAC E142</td>
<td>3He A1, A2, g1, g2</td>
</tr>
<tr>
<td>SLAC E143</td>
<td>proton and deuteron A_par, A_perp</td>
</tr>
<tr>
<td>SLAC E154</td>
<td>3He A-par, A_perp</td>
</tr>
<tr>
<td>SLAC E155</td>
<td>proton and deuteron A_parallel (longitudinal), A_perp (transverse), A1, A2, g1, g2</td>
</tr>
<tr>
<td>SLAC E155x</td>
<td>proton and deuteron A_perp (transverse), A2, g2</td>
</tr>
<tr>
<td>SLAC E80/E130</td>
<td>proton A_parallel</td>
</tr>
<tr>
<td>SMC</td>
<td>proton and deuteron A1, g1</td>
</tr>
</tbody>
</table>
Add TMD data into this database
Framework
 Written in C++
uses ROOT
Allows many users to contribute
Global analysis:

Model ansatz for TMDs, Initial set of parameters

Evolve TMDs to relevant scale

Calculate observables and $\chi^2$

$\chi^2$ minimum?

- Yes!
- No!

Change parameters
Global analysis:

1. Model ansatz for TMDs, Initial set of parameters
2. Evolve TMDs to relevant scale
3. Calculate observables and $\chi^2$

Parametrization must be flexible but economical in terms of number of parameters.

- $\chi^2$ minimum?
  - Yes!
  - No!
  - Change parameters
Global analysis:

- Model ansatz for TMDs, Initial set of parameters
- Evolve TMDs to relevant scale
- Calculate observables and $\chi^2$
- $\chi^2$ minimum?
  - Yes!
  - No!

Parametrization must be flexible but economical in terms of number of parameters.
Evolution should be done fast.
Global analysis:

- Model ansatz for TMDs, Initial set of parameters
- Evolve TMDs to relevant scale
  - Parametrization must be flexible but economical in terms of number of parameters
  - Evolution should be done fast
  - Number of observables \( \sim 1000 \)’s to minimize \( \chi^2 \)
- Calculate observables and \( \chi^2 \)
  - \( \chi^2 \) minimum?
    - No!
    - Yes!

\( \chi^2 \) minimum?
Global analysis:

- Model ansatz for TMDs, Initial set of parameters
- Evolve TMDs to relevant scale
- Calculate observables and $\chi^2$
- $\chi^2$ minimum?

Yes!  
No!

- Evolution should be done fast
- Parametrization must be flexible but economical in terms of number of parameters
- Number of observables ~1000's to minimize $\chi^2$

Calculation of TMD expressions is very time consuming!
Why?

Structure functions are convolutions of unobserved momenta:

\[ F \sim \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_h) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp) \]
Why?

Structure functions are convolutions of unobserved momenta:

\[ F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_h) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp) \]

Observed in experiment

No analogue of Mellin transform to help to perform this convolution found yet!
Strengthen the case of JLAB 12 and ELECTRON ION COLLIDER

EIC  JLab 12

$\mathbf{JLab\ 12\ and\ Electron\ Ion\ Collider}$

A. Prokudin (2012) contribution to EIC white paper
CONCLUSIONS

GPDs

PDFs

TMDs

Experiment