Measurement of $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$ with Bubble Chamber and Bremsstrahlung Beam at Jefferson Lab Injector

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OUTLINE

• Nucleosynthesis and the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Reaction
• Time Reversal Reaction: $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$
• The Bubble Chamber
• Experimental Setup at Jefferson Lab Injector
• Bremsstrahlung Beam and Penfold-Leiss Unfolding
• Statistical and Systematic Errors
• Backgrounds and Ion Energy Distributions
• Summary and Outlook
Relative Abundance of Elements by Weight

Universe
- Hydrogen: 73.9%
- Helium: 24.0%
- Oxygen: 1.0%
- Carbon: 0.5%
- Other: 0.6%

Human Body
- Oxygen: 61%
- Carbon: 23%
- Hydrogen: 10%
- Nitrogen: 2.6%
- Calcium: 1.1%
- Phosphorus: 1.1%
- Other: 0.9%

This region is bypassed by 3α process.
Nucleosynthesis

- **Big Bang Nucleosynthesis:** quark–gluon plasma → p, n, He
- **Stellar Nucleosynthesis:** H burning, He burning, NCO cycle
- **Supernovae Nucleosynthesis:** Si burning
- **Cosmic Ray Spallation**
**Stellar Helium Burning**

- **Helium Reactions:**
  
  I. \( \alpha + \alpha \leftrightarrow ^8\text{Be} \)  
  \((Q = -0.092 \text{ MeV, } T_{1/2} \approx 10^{-16} \text{ s}) \)
  
  II. \( \alpha + ^8\text{Be} \rightarrow ^{12}\text{C} + \gamma \)  
  \((Q = +7.367 \text{ MeV, Hoyle State} = 7.654 \text{ MeV}) \)
  
  III. \( \alpha + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma \)  
  \((\text{slow, otherwise no } ^{12}\text{C remains}) \)
  
  IV. \( \alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne} + \gamma \)  
  \((\text{very slow – due to parity conservation}) \)

- \( \alpha + ^{12}\text{C} \) burns at very small cross section \( \sigma \approx 10^{-17} \text{ barn} \) \((10^{-41} \text{ cm}^2) \)

- Currently, reaction rate error is large \((\pm 35\%) \)

- **Goal <\pm 10\%**

- Thermonuclear reaction rate involving two nuclei is:

  \[
  R = \sqrt{\frac{8}{\pi m (k_B T)^3}} \int_0^\infty E \sigma_{\text{tot}}(E) e^{-\frac{E}{k_B T}} dE
  \]

  Only narrow energy range is relevant (Gamow Peak)
THE $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Reaction

- The *holy grail* of nuclear astrophysics

Affects the synthesis of most of the elements of the periodic table

Sets the $N(^{12}\text{C})/N(^{16}\text{O}) \approx 0.4$ ratio in the universe

Determines the minimum mass a star requires to become a supernova

The Gamow Peak (Window)

- Narrow energy range where thermonuclear reactions is most likely to occur in stellar plasma is a product of two distributions:
  
  I. Maxwell-Boltzmann energy distribution with $e^{-E/k_B T}$
  
  II. Penetration through Coulomb barrier with $e^{-b/E^{1/2}}$

- For $\alpha + ^{12}C$, and stellar $T=200 \times 10^6$ K:
  
  - Gamow Peak, $E_0 \approx 300$ keV, Width $\approx 50$ keV (in Center-of-Mass (CM) of $\alpha + ^{12}C$ system)
  
  - Maximum of Maxwell–Boltzmann energy distribution, $k_B T = 17$ keV
\(\alpha + ^{12}\text{C} \text{ Reaction}\)

- \(\alpha (J^{\pi}=0^+) + ^{12}\text{C} (J^{\pi}=0^+)\) cross section, \(\sigma(E_0)\), is dominated by \(p\)-wave (E1) and \(d\)-wave (E2) radiative capture to \(^{16}\text{O}\) ground state (\(J^{\pi}=0^+)\)

- Two bound states, at 6.92 MeV (\(J^{\pi}=2^+)\) and 7.12 MeV (\(J^{\pi}=1^-\)), with sub–threshold resonances at \(E_R=-0.245\) and -0.045 MeV, provide most of \(\sigma(E_0)\) through their finite widths

- Distinguish E1 and E2 by measuring \(\gamma\)–angular distributions
Heroic efforts in search of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Previous Experiments:

A. Direct Measurements:
   I. Helium ions on carbon target: $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$
   II. Carbon ions on helium gas: $^4\text{He}^{(12}\text{C}, \gamma)^{16}\text{O}$ or $^4\text{He}^{(12}\text{C},^{16}\text{O})\gamma$ (Schürmann)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Beam Current (mA)</th>
<th>Target (nuclei/cm$^2$)</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redder</td>
<td>0.7</td>
<td>$^{12}\text{C}, 3\times 10^{18}$</td>
<td>900</td>
</tr>
<tr>
<td>Ouellet</td>
<td>0.03</td>
<td>$^{12}\text{C}, 5\times 10^{18}$</td>
<td>1950</td>
</tr>
<tr>
<td>Roters</td>
<td>0.02</td>
<td>$^4\text{He}, 1\times 10^{19}$</td>
<td>5000</td>
</tr>
<tr>
<td>Kunz</td>
<td>0.5</td>
<td>$^{12}\text{C}, 3\times 10^{18}$</td>
<td>700</td>
</tr>
<tr>
<td>EUROGAM</td>
<td>0.34</td>
<td>$^{12}\text{C}, 1\times 10^{19}$</td>
<td>2100</td>
</tr>
<tr>
<td>GANDI</td>
<td>0.6 (?)</td>
<td>$^{12}\text{C}, 2\times 10^{18}$</td>
<td>?</td>
</tr>
<tr>
<td>Schürmann</td>
<td>0.01</td>
<td>$^4\text{He}, 4\times 10^{17}$</td>
<td>?</td>
</tr>
<tr>
<td>Plag</td>
<td>0.005</td>
<td>$^{12}\text{C}, 6\times 10^{18}$</td>
<td>278</td>
</tr>
</tbody>
</table>

B. Indirect Measurements:
   I. $\beta$–delayed $\alpha$ decay of $^{16}\text{N}$ ($J^\pi=2^-, T_{1/2}=7.13$ s, BR=0.12%)
      $^{16}\text{N} \rightarrow \beta^- + ^{16}\text{O}^* \ (J^\pi=1^-) \rightarrow \alpha + ^{12}\text{C}$
ASTROPHYSICAL S-FACTOR $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Define $S$-Factor to remove both $1/E$ dependence of nuclear cross sections and Coulomb barrier transmission probability:

$$S \equiv E_{CM} \sigma(\alpha, \gamma) e^{2\pi \eta}$$

$$\eta = \frac{1}{137} Z_\alpha Z_{^{12}C} \sqrt{\frac{m_{^{12}C\alpha}}{2E_{CM}}}$$

<table>
<thead>
<tr>
<th>Author</th>
<th>$S_{\text{tot}}(300)$ (keV b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hammer (2005)</td>
<td>162±39</td>
</tr>
<tr>
<td>Kunz (2001)</td>
<td>165±50</td>
</tr>
</tbody>
</table>

R-matrix Extrapolation to stellar helium burning at $E = 300$ keV
**TIME REVERSAL REACTION**

Stellar helium burning at $E = 300$ keV
**Reciprocity Relation: \((\gamma, \alpha)\) and \((\alpha, \gamma)\)**

- **A(\(\alpha, \gamma\))B:**

\[
\sigma_{B\gamma}^{j\rightarrow i}(E_\gamma) = \frac{(2J_i + 1)(2J_\alpha + 1)}{2J_j + 1} \frac{m_{A\alpha}c^2E_{A\alpha}}{E_\gamma^2} \sigma_{A\alpha}^{i\rightarrow j}(E_{A\alpha})
\]

- **\(m_{A\alpha}c^2 = \frac{M^{(12C)} \cdot M(\alpha)}{M^{(12C)} + M(\alpha)} = 2796 \text{ MeV} \)**
  \(J_i = 0, J_j = 0, J_\alpha = 0\)

- **\(E_{A\alpha} = E_{CM}\)**

- **\(Q = m_A + m_\alpha - m_B = +7.162 \text{ MeV}\)**

- **\(E_\gamma \cong E_{CM} + Q\)**

- **\(\sigma(\gamma, \alpha)\) is over two orders of magnitude larger than \(\sigma(\alpha, \gamma)\)**
**NEW APPROACH: REVERSAL REACTION + BUBBLE CHAMBER**

- Extra gain (factor of 100) by measuring time reversal reaction

- Target density up to $10^4$ higher than conventional targets. Number of $^{16}$O nuclei = $3.5 \times 10^{22}$ /cm$^2$ (3.0 cm cell)

- Measures total cross section $\sigma_{tot}$ (or $S_{tot}$)

- Solid Angle and Detector Efficiency = 100%

- Electromagnetic debris (electrons and gammas, or positrons) do NOT trigger nucleation (detector is insensitive to $\gamma$-rays by at least 1 part in $10^{11}$).

\[ \gamma + ^{16}\text{O} \rightarrow ^{12}\text{C} + \alpha \]
THE BUBBLE CHAMBER

- Donald Glaser won Nobel Prize for inventing chamber to detect particles (1960)
- Now being used in Dark Matter Search Experiments: COUPP, PICASSO, SIMPLE

Superheat Preparation:
- Liquid is pressurized at ambient temperature (1 to 2)
- Then pressure is kept constant while temperature is increased to above boiling point (2 to 3)
- Finally pressure is slowly released while keeping temperature constant (3 to 4)
- At this point (4), still liquid but now superheated

Bubble Formation:
- Particle energy loss will induce vaporization
- Resultant vapor bubble is observable either visibly or audibly
- Bubble growth is captured by a digital camera
- Pressure is increased (4 to 3) to quench bubble. It takes about a second for liquid to return to a stable state
- Superheat is restored by releasing pressure again (3 to 4), and cycle is repeated for each bubble event
**Bubble Growth and Quenching**

$^{19}$F($\gamma,\alpha$)$^{15}$N event in C$_4$F$_{10}$ at HIGS

100 Hz Digital Camera: $\Delta t = 10$ ms
ACOUSTIC SIGNAL DISCRIMINATION

I. Bubble growth produces an audible click which is recorded by piezo-electric transducers

II. Neutron Events:
   I. $^{17}\text{O}(\gamma,n)^{16}\text{O}$
   II. Neutron–nucleus elastic scattering: $^{16}\text{O}(n,n)$, $^{14}\text{N}(n,n)$
Ions $^{16}\text{O}$ or $^{14}\text{N}$ will generate a single bubble

III. Alpha Events:
   I. $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$
   II. $^{17}\text{O}(\gamma,\alpha)^{13}\text{C}$
   III. $^{18}\text{O}(\gamma,\alpha)^{14}\text{C}$
Ions $^{12}\text{C}+\alpha$ or $^{13}\text{C}+\alpha$ or $^{14}\text{C}+\alpha$ will generate a combined multi-bubble

Suppress neutron events by 100 using acoustic signal

Higher Pitch

COUPP, FNAL, courtesy of A. Sonnenschein
N₂O (Laughing Gas) Bubble Chamber

T = -5°C
P = 60 atm

First $\gamma + O \rightarrow \alpha + C$ bubble
HIGS, April 2013
I. For bubble formation, particle must be over thresholds in both $E$ and $dE/dx$

$$E \geq E_c = \frac{4}{3} \pi R_c^3 (\rho h + P_i) + 4 \pi R_c^2 \left( s - T \frac{\partial s}{\partial T} \right)$$

II. Only bubbles with $r > R_c$ grow to be macroscopic

$$R_c = 2s / (P_v - P_l)$$

$s$: Surface tension

III. Bubble requires minimum deposited energy ($E_c$) within minimum distance $L_c$ ($= aR_c$, 10s of nm to a few µm)

$$\frac{dE}{dx} > \left( \frac{dE}{dx} \right)_c = \frac{E_c}{aR_c}$$

$a$: free parameter (to determined experimentally)
Efficiency Curve

$N_2O$ thresholds
Superheat = 3.3 °C

$\gamma + ^{16}O$  $\gamma + ^{14}N$ and $\gamma + ^{16}O$

$N_2O$ efficiency curve, HIGS April 2013, $E_\gamma = 9.7$ MeV
EXPERIMENTAL SETUP AT JLAB INJECTOR

- BCM
- 5 MeV Dipole
- 5D Spectrometer
- 2D Spectrometer
- Mott Polarimeter
- Bubble Chamber location
**Beamline**

Bubble Chamber at HIGS April 2013

**Electron Beam Requirements at Radiator**

- Beam Kinetic Energy, (MeV): 7.9–8.5
- Beam Current (µA): 0.01–100
- Absolute Beam Energy Uncertainty: <0.1%
- Relative Beam Energy Uncertainty: <0.02%
- Energy Resolution (Spread), $\sigma_T/T$: <0.06%
- Beam Size, $\sigma_{x,y}$ (mm): 1–2

**Photon Beam Entrance**
**SCHEMATICS**

- Power deposited in radiator (100 μA and 8.5 MeV):
  I. 0.02 mm: Energy loss = 21 keV, P = 2.1 W

- Pure Copper and Aluminum (high neutron threshold):
  I. $^{63}$C($\gamma$,n) threshold = 10.86 MeV
  II. $^{27}$Al($\gamma$,n) threshold = 13.06 MeV

*Use GEANT4 to design this region*
MEASURING ABSOLUTE BEAM ENERGY

5 MeV Dipole

Electron Beam Momentum

\[ p = \frac{\int B dl}{\theta} \]
I. Jay Benesch designed and is now working with Engineering to fabricate a more uniform and higher field dipole

II. New Hall Probe: 0.01% accuracy, resolution to 2 ppm, and a temperature stability of 10 ppm/°C

III. Better shielding of Earth’s and other stray magnetic fields

IV. Additional goal: Relative beam energy uncertainty $<0.02\%$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Term</th>
<th>Now</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole – linearity</td>
<td>$\delta B/B$</td>
<td>0.25%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Dipole – spatial</td>
<td>$\delta BL/BL$</td>
<td>0.10%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Dipole – reproduce</td>
<td>$\delta B/B$</td>
<td>0.10%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Dipole – power supply</td>
<td>$\delta I/I$</td>
<td>0.20%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Position – surveys</td>
<td>$\delta \theta/\theta$</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Position – BPM calibration</td>
<td>$\delta \theta/\theta$</td>
<td>0.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Stray magnetic field</td>
<td>$\delta \theta/\theta$</td>
<td>0.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Total</td>
<td>$\delta P/P$</td>
<td>0.36%</td>
<td>$&lt;0.10%$</td>
</tr>
</tbody>
</table>
Use both GEANT4 and FLUKA to calculate Bremsstrahlung spectra (we will not measure Bremsstrahlung spectra)

Monte Carlo simulation of Bremsstrahlung at radiotherapy energies is well studied, accuracy: ±5%

$^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$ is ideal case for Bremsstrahlung beam and Penfold–Leiss Unfolding:

I. Very steep; only photons near endpoint contribute to yield
II. No-structure (resonances)
GEANT4 SIMULATION

- Both GEANT4 and FLUKA use models that calculate wrong photo-nuclear cross sections. Both do not allow for user’s cross sections. What to do?
  
  I. Use GEANT4 and FLUKA to produce the photon spectra impinging on the superheated liquid.
  II. Fold the above photon spectra with our cross sections in stand-alone codes.

- Use GEANT4 to design radiator, collimator, and dumps

- Geometry in GEANT4:
**Penfold-Leiss Cross Section Unfolding**

- Measure yields at: \( E = E_1, E_2, \ldots, E_n \) where, 
  \( E_i - E_{i-1} = \Delta, i = 2, n \)

\[
Y(E_i) = \int_{th}^{E_i} n_\gamma(E_i, k)\sigma(k)dk \approx \sum_{j=1}^{i} N_{\gamma}(E_i, \Delta, E_j)\sigma(E_j)
\]

Volterra Integral Equation of First Kind

- The solution can be written in two forms:

\[
\sigma_i = \frac{1}{N_{ii}} \left[ y_i - \sum_{j=1}^{i-1}(N_{ij}\sigma_j) \right]
\]

- Or, Matrix form:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{bmatrix} = \begin{bmatrix}
N_{\gamma,11} & 0 & \cdots & 0 \\
N_{\gamma,21} & N_{\gamma,22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
N_{\gamma,n1} & N_{\gamma,n2} & \cdots & N_{\gamma,nn}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_n
\end{bmatrix}
\]

\[
[\sigma] = [N]^{-1} \bullet [Y]
\]

Method of Quadratures: numerical solution of integral equation based on replacement of integral by finite sum.
STATISTICAL ERROR PROPAGATION

• Note:
  \[ \frac{dy_i}{y_i} = \frac{1}{\sqrt{y_i}} \]
  \[ \frac{dN_{ij}}{N_{ij}} = \frac{1}{\sqrt{N_{ij}}} \approx 0 \]

  \[ dy_i = \sqrt{y_i} \]
  \[ dy_i = \sqrt{y_i + 2y_i^{bg}} \]

• With:
  \[ [B] = [N]^{-1} \]
  \[ [\sigma] = [B] \bullet [Y] \]

• Then:
  \[ [d\sigma^2] = [B] \bullet [dY^2] \bullet [B]^T \]

In case of background Subtraction
\[ (d\sigma_i)^2 = \frac{1}{N_{ii}^2} \left[ dy_i^2 + \sum_{j=1}^{i-1} \left( N_{ij} d\sigma_j \right)^2 + \sum_{k=1}^{i-1} \sum_{l=1}^{i-1} N_{ik} \cov(\sigma_k, \sigma_l) N_{il} \right] \]

For mono-chromatic photon beam

\[ \left( \frac{d\sigma_i}{\sigma_i} \right)^2 = \left( \frac{dy_i}{y_i} \right)^2 = \frac{1}{y_i} \]
RESULTS

I. Radiator thickness = 0.02 mm
II. Bubble Chamber thickness = 3.0 cm, number of $^{16}$O nuclei = $3.474 \times 10^{22}$ /cm$^2$
III. Background subtraction of $^{18}$O($\gamma, \alpha$)$^{14}$C

$$[N] = \begin{bmatrix} 3.267e14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9.782e13 & 6.439e13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5.013e13 & 3.858e13 & 2.539e13 & 0 & 0 & 0 & 0 & 0 \\ 1.494e13 & 1.236e13 & 9.514e12 & 6.258e12 & 0 & 0 & 0 & 0 \\ 8.540e12 & 7.369e12 & 6.097e12 & 4.692e12 & 3.086e12 & 0 & 0 & 0 \\ 3.801e12 & 3.370e12 & 2.908e12 & 2.406e12 & 1.852e12 & 1.217e12 & 0 & 0 \\ 2.075e12 & 1.875e12 & 1.663e12 & 1.435e12 & 1.187e12 & 9.137e11 & 6.004e11 & 0 \end{bmatrix}$$

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Beam Current (µA)</th>
<th>Time (hour)</th>
<th>$y_i$</th>
<th>$dy_i$ (no bg)</th>
<th>$dy_i/y_i$ (no bg, %)</th>
<th>$dy_i$ (with bg)</th>
<th>$dy_i/y_i$ (with bg, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>100</td>
<td>100</td>
<td>545</td>
<td>23</td>
<td>4.2</td>
<td>134</td>
<td>24.6</td>
</tr>
<tr>
<td>8.0</td>
<td>100</td>
<td>20</td>
<td>581</td>
<td>24</td>
<td>4.1</td>
<td>77</td>
<td>13.3</td>
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<tr>
<td>8.1</td>
<td>80</td>
<td>10</td>
<td>852</td>
<td>29</td>
<td>3.4</td>
<td>60</td>
<td>7.0</td>
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<tr>
<td>8.2</td>
<td>20</td>
<td>10</td>
<td>634</td>
<td>25</td>
<td>3.9</td>
<td>40</td>
<td>6.3</td>
</tr>
<tr>
<td>8.3</td>
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<td>812</td>
<td>28</td>
<td>3.4</td>
<td>39</td>
<td>4.8</td>
</tr>
<tr>
<td>8.4</td>
<td>4</td>
<td>10</td>
<td>746</td>
<td>27</td>
<td>3.6</td>
<td>36</td>
<td>4.8</td>
</tr>
<tr>
<td>8.5</td>
<td>2</td>
<td>10</td>
<td>763</td>
<td>28</td>
<td>3.7</td>
<td>32</td>
<td>4.2</td>
</tr>
</tbody>
</table>
**Systematic Error Propagation**

- For absolute beam energy uncertainty of $\delta E = 0.1\%$ and zero relative beam energy uncertainty:

$$\frac{dy_i}{y_i} = \frac{y_i(E_i + \delta E) - y_i(E_i)}{y_i(E_i)}$$

$$\frac{dN_{ij}}{N_{ij}} = \frac{N_{ij}(E_i + \delta E) - N_{ij}(E_i)}{N_{ij}(E_i)}$$

$$E_0 = 7.8 + \delta E$$

$$E_i = E_0 + i\Delta$$

<table>
<thead>
<tr>
<th>$E_i$ (MeV)</th>
<th>$dy_i/y_i$ (%)</th>
<th>$d\sigma_i/\sigma_i$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>12.5</td>
<td>12.6</td>
</tr>
<tr>
<td>8.0</td>
<td>10.8</td>
<td>10.5</td>
</tr>
<tr>
<td>8.1</td>
<td>9.3</td>
<td>9.1</td>
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<td>8.2</td>
<td>8.0</td>
<td>7.1</td>
</tr>
<tr>
<td>8.3</td>
<td>7.0</td>
<td>6.3</td>
</tr>
<tr>
<td>8.4</td>
<td>6.3</td>
<td>5.8</td>
</tr>
<tr>
<td>8.5</td>
<td>5.6</td>
<td>5.2</td>
</tr>
</tbody>
</table>

- Accounted for $dN_{ij}$ due to energy error when calculating $dy_i$.
\[
\frac{\delta E}{i \Delta} \approx \frac{dN_{ij}}{N_{ij}}
\]

\[
\begin{bmatrix}
0.100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.058 & 0.050 & 0 & 0 & 0 & 0 & 0 \\
0.041 & 0.039 & 0.033 & 0 & 0 & 0 & 0 \\
0.031 & 0.031 & 0.029 & 0.025 & 0 & 0 & 0 \\
0.025 & 0.025 & 0.025 & 0.023 & 0.020 & 0 & 0 \\
0.021 & 0.021 & 0.021 & 0.021 & 0.020 & 0.017 & 0 \\
0.018 & 0.018 & 0.018 & 0.018 & 0.018 & 0.017 & 0.022
\end{bmatrix}
\]

- With:

\[
[B] = [N]^{-1}
\]

\[
[\sigma] = [B] \bullet [Y]
\]

- Then:

\[
[d \sigma^2] = [B] \bullet \left( [dY^2] + [dN^2] \bullet [\sigma^2] \right) \bullet [B]^T
\]
- Where:

Note: Correlation Coefficient ($\rho_{ij}$) = 1

\[
[dY^2] = \begin{pmatrix}
(dy_1)^2 & dy_1 dy_2 & \cdots & dy_1 dy_n \\
(dy_2) & (dy_2)^2 & \cdots & dy_2 dy_n \\
\vdots & \vdots & \ddots & \vdots \\
(dy_n) & dy_n dy_1 & \cdots & (dy_n)^2 \\
\end{pmatrix}
\]

\[
[d\sigma^2] = \begin{pmatrix}
d\sigma_1^2 & \text{cov}(\sigma_1, \sigma_2) & \cdots & \text{cov}(\sigma_1, \sigma_n) \\
\text{cov}(\sigma_2, \sigma_1) & d\sigma_2^2 & \cdots & \text{cov}(\sigma_2, \sigma_n) \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}(\sigma_n, \sigma_1) & \text{cov}(\sigma_n, \sigma_2) & \cdots & d\sigma_n^2 \\
\end{pmatrix}
\]

\[
[dN^2] = \begin{pmatrix}
(dN_{11})^2 & 0 & \cdots & 0 \\
(dN_{21})^2 & (dN_{22})^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
(dN_{n1})^2 & (dN_{n2})^2 & \cdots & (dN_{nn})^2 \\
\end{pmatrix}
\]

\[
[\sigma^2] = \begin{pmatrix}
\sigma_1^2 & 0 & \cdots & 0 \\
0 & \sigma_2^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_n^2 \\
\end{pmatrix}
\]

\[
\text{var}(y_i, y_i) = (dy_i)^2 \\
\text{cov}(y_i, y_j) = \rho_{ij} dy_i dy_j
\]

No energy-to-energy change in systematic error
\[(d\sigma_i)^2 \approx \frac{1}{N_{ii}^2} \left[ dy_i^2 - 2dy_i \sum_{j=1}^{i-1} N_{ij} d\sigma_j \right]\]

\[+ \sum_{j=1}^{i-1} \left( N_{ij} d\sigma_j \right)^2 + \sum_{k=1}^{i-1} \sum_{l=1}^{i-1} N_{ik} \text{cov}(\sigma_k, \sigma_l) N_{il}\]

\[+ \sum_{j=1}^{i-1} \left( dN_{ij} \sigma_j \right)^2 + \left( dN_{ii} \sigma_i \right)^2 \]
# Other Systematic Errors

<table>
<thead>
<tr>
<th>Error Description</th>
<th>Error Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current, $\delta I/I$</td>
<td>3%</td>
</tr>
<tr>
<td>Photon Flux, $\delta \phi/\phi$</td>
<td>5%</td>
</tr>
<tr>
<td>Radiator Thickness, $\delta R/R$</td>
<td>3%</td>
</tr>
<tr>
<td>Bubble Chamber Thickness, $\delta T/T$</td>
<td>3%</td>
</tr>
<tr>
<td>Bubble Chamber Efficiency, $\epsilon$</td>
<td>5%</td>
</tr>
</tbody>
</table>

Then:

\[
(dy_i)^2 = (dy_i (\delta E))^2 + \left[ \left( \frac{\delta I}{I} \right)^2 + \left( \frac{\delta R}{R} \right)^2 + \left( \frac{\delta T}{T} \right)^2 + \epsilon^2 \right] y_i^2
\]

\[
(dN_{ij})^2 = \left( \frac{\delta \phi}{\phi} \right)^2 N_{ij}^2
\]
<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Cross Section (nb)</th>
<th>Stat Error (no bg, %)</th>
<th>Stat Error (with bg, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>0.046</td>
<td>4.4</td>
<td>24.5</td>
</tr>
<tr>
<td>8.0</td>
<td>0.185</td>
<td>6.0</td>
<td>20.7</td>
</tr>
<tr>
<td>8.1</td>
<td>0.58</td>
<td>6.3</td>
<td>14.7</td>
</tr>
<tr>
<td>8.2</td>
<td>1.53</td>
<td>8.2</td>
<td>13.8</td>
</tr>
<tr>
<td>8.3</td>
<td>3.49</td>
<td>9.1</td>
<td>13.3</td>
</tr>
<tr>
<td>8.4</td>
<td>7.2</td>
<td>10.6</td>
<td>13.8</td>
</tr>
<tr>
<td>8.5</td>
<td>13.6</td>
<td>12.2</td>
<td>14.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Cross Section (nb)</th>
<th>Sys Error (Energy, %)</th>
<th>Sys Error (Total, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>0.046</td>
<td>12.5</td>
<td>15.3</td>
</tr>
<tr>
<td>8.0</td>
<td>0.185</td>
<td>10.2</td>
<td>13.5</td>
</tr>
<tr>
<td>8.1</td>
<td>0.58</td>
<td>8.3</td>
<td>12.2</td>
</tr>
<tr>
<td>8.2</td>
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<td>3.49</td>
<td>6.0</td>
<td>10.7</td>
</tr>
<tr>
<td>8.4</td>
<td>7.2</td>
<td>5.3</td>
<td>10.5</td>
</tr>
<tr>
<td>8.5</td>
<td>13.6</td>
<td>4.7</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Note: Absolute systematic errors do not get magnified in PL Unfolding
JLab Projected $^{12}$C($\alpha,\gamma$)$^{16}$O S-Factor

- Statistical Error: dominated by background subtraction from $^{18}$O($\gamma,\alpha$)$^{14}$C (depletion = 5,000)

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Gamma Energy (MeV)</th>
<th>$E_{CM}$ (MeV)</th>
<th>Cross Section (nb)</th>
<th>$S_{tot}$ Factor (keV b)</th>
<th>Stat Error (%)</th>
<th>Sys Error (Total, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>7.85</td>
<td>0.69</td>
<td>0.046</td>
<td>62.2</td>
<td>24.5</td>
<td>15.3</td>
</tr>
<tr>
<td>8.0</td>
<td>7.95</td>
<td>0.79</td>
<td>0.185</td>
<td>48.7</td>
<td>20.7</td>
<td>13.5</td>
</tr>
<tr>
<td>8.1</td>
<td>8.05</td>
<td>0.89</td>
<td>0.58</td>
<td>41.8</td>
<td>14.7</td>
<td>12.2</td>
</tr>
<tr>
<td>8.2</td>
<td>8.15</td>
<td>0.99</td>
<td>1.53</td>
<td>35.5</td>
<td>13.8</td>
<td>11.4</td>
</tr>
<tr>
<td>8.3</td>
<td>8.25</td>
<td>1.09</td>
<td>3.49</td>
<td>32.0</td>
<td>13.3</td>
<td>10.7</td>
</tr>
<tr>
<td>8.4</td>
<td>8.35</td>
<td>1.19</td>
<td>7.2</td>
<td>28.8</td>
<td>13.8</td>
<td>10.5</td>
</tr>
<tr>
<td>8.5</td>
<td>8.45</td>
<td>1.29</td>
<td>13.6</td>
<td>26.3</td>
<td>14.8</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Bubble Chamber experiment measures total S-Factor, $S_{E1} + S_{E2}$
I. Background from oxygen isotopes and nitrogen in N\textsubscript{2}O:
   - \(^{18}\text{O}(\gamma,\alpha)^{14}\text{C}\)
   - \(^{17}\text{O}(\gamma,\alpha)^{13}\text{C}\)
   - \(^{14}\text{N}(\gamma,p)^{13}\text{C}\)

* Natural Abundance:
  I. \(^{17}\text{O}: 0.038\%\)
  II. \(^{18}\text{O}: 0.205\%\)

* Expected Rates:
  I. \(^{17}\text{O}(\gamma,\alpha)^{13}\text{C}, \text{depletion}=5,000\)
  II. \(^{18}\text{O}(\gamma,\alpha)^{14}\text{C}, \text{depletion}=5,000\)
  III. \(^{14}\text{N}(\gamma,p)^{13}\text{C}, \text{Chamber eff.}=10^{-8}\)
II. Background from:
   – $^{17}\text{O}(\gamma,n)^{16}\text{O}$ and secondary (n,n) neutron–nucleus elastic scattering

III. Background from Chamber glass:
   – Neutron–nucleus elastic scattering from $^{29}\text{Si}(\gamma,n)^{28}\text{Si}$

IV. Cosmic–ray background:
   – $\mu^+$–nuclear
   – neutron–nuclear elastic scattering

➢ Reject neutron events using acoustic signal (100 suppression factor)
ION ENERGY DISTRIBUTIONS

- Use depleted \( \text{N}_2\text{O} \): 
  1. \( ^{17}\text{O} \) depletion = 5,000 
  2. \( ^{18}\text{O} \) depletion = 5,000

- Suppress background with Bubble Chamber thresholds

\[
E_{CM} \approx E_{\gamma} - Q \\
E_{CM} = T_{\alpha} + T_{C}
\]

\[
T_{\alpha,\text{lab}} \approx \frac{m_c}{m_\alpha + m_c} E_{CM} \\
T_{C,\text{lab}} \approx \frac{m_\alpha}{m_\alpha + m_c} E_{CM}
\]

- Threshold Efficiency (function of superheat):

<table>
<thead>
<tr>
<th>Particle</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^\pm )</td>
<td>(&lt;10^{-11})</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>(&lt;10^{-11})</td>
</tr>
<tr>
<td>( ^{14}\text{N}(\gamma,p)^{13}\text{C} )</td>
<td>(&lt;10^{-8})</td>
</tr>
</tbody>
</table>
SUPERHEATED TARGETS

I. List of superheated liquids to be used in experiment:

<table>
<thead>
<tr>
<th>N₂O Targets</th>
<th>¹⁶O</th>
<th>¹⁷O</th>
<th>¹⁸O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Target</td>
<td>99.757%</td>
<td>0.038%</td>
<td>0.205%</td>
</tr>
<tr>
<td>¹⁶O Target</td>
<td></td>
<td>Depleted &gt; 5,000</td>
<td>Depleted &gt; 5,000</td>
</tr>
<tr>
<td>¹⁷O Target</td>
<td></td>
<td>Enriched &gt; 80%</td>
<td>&lt;1.0%</td>
</tr>
<tr>
<td>¹⁸O Target</td>
<td></td>
<td>&lt;1.0%</td>
<td>Enriched &gt; 80%</td>
</tr>
</tbody>
</table>

II. Readout:

I. Fast Digital Camera

II. Acoustic Signal to discriminate between neutron and alpha events
SUMMARY AND OUTLOOK

• Test N₂O Bubble Chamber at HIGS (Summer 2014)
• Measure cross sections of \(^{18}\text{O}(\gamma, \alpha)^{14}\text{C}\) and \(^{17}\text{O}(\gamma, \alpha)^{13}\text{C}\) at HIGS (Fall 2014)
• Test Bubble Chamber at JLab (October 2014, January 2015)
• Run depleted N₂O bubble chamber at JLab to measure \(^{16}\text{O}(\gamma, \alpha)^{12}\text{C}\)

• Beam issues:
  – Design radiator, collimator, and dumps with GEANT4
  – Simulate photon spectra with GEANT4 and FLUKA
  – Deliver 8.5 MeV K.E. beam to 5D Spectrometer with <0.1% absolute energy uncertainty

• Bubble Chamber issues:
  – Study acoustic signal and measure neutron events suppression factor
  – Deadtime measurements: use laser shutter to stop beam while chamber is not ready
  – Measure O-isotopes depletion

• Background tests:
  – Measure cosmic-ray background
  – Study chamber thresholds efficiency vs. superheat and measure \(\gamma\)-rays suppression factor
**CO$_2$ SUPERHEATED LIQUID?**

- Similar Bubble Chamber operational parameters as N$_2$O
- Natural Abundance: $^{13}$C: 1.07%
- Depletion: $^{13}$C depletion=1,000
- $^{13}$C($\gamma$,n)$^{12}$C Background

For comparison, $^{17}$O($\gamma$,n)$^{16}$O

- $^{12}$C($\gamma$,2$\alpha$)$\alpha$ Background
**Water Superheated Liquid?**

- Etching of glass vessel by superheated H₂O
  - T = 250°C
  - P = 75 atm

- Background from secondary neutron–nucleus elastic scattering by neutrons from d(γ,n)p
Rolfs and Rodney, 1988