Resonances in experiments
(e.g., meson photo/electro-production)
Resonances in experiments
(e.g., meson photo/electro-production)

$N^*(1440)$
the Roper: $\Gamma \sim 350$ MeV

short-lived state, decays in $\sim 10^{-23}s$
Resonances in experiments
(e.g., meson photo/electro-production)

$N^*(1440)$
the Roper: $\Gamma \sim 350$ MeV
Resonances in experiments
(e.g., meson photo/electro-production)

must be reconstructed from its byproducts

$N^*(1440)$
the Roper: $\Gamma \sim 350 \text{ MeV}$

particles that hit the detector
Resonances in experiments
(e.g., rare weak decays)

“Flavor changing neutral currents”

“3.7σ tensions”

2013
Resonances in experiments
(e.g., exotic states)

\[ J/\Psi \rightarrow P_c^+ (4380) \]
\[ P_c^+ (4450) \]
Resonances in experiments (e.g., exotic states)
Resonant matrix elements
(e.g., enigmatic states)

$\sigma / f_0(500), f_0(980), \ldots$
Why are resonances important?
Why are resonances important?
Resonances - Intuitive picture

[Incoming states]

[Resonating state]

[Outgoing states]
Scattering theory 101

no scattering

\[
\psi \sim e^{ipz}
\]

\( p = \) relative momentum

scattering

\[
\psi \sim e^{ipz} + \frac{e^{i\delta} \sin \delta}{p} e^{ipr}
\]

\( \delta = \) scattering phase shift

“encode all physics”
A pseudo-quantitative definition
(bump in an amplitude - e.g., ππ scattering in η-channel)

\[ M_1 = \frac{8\pi E_{\text{cm}}}{p} \frac{1}{\cot \delta_1 - i} \]

Protopopescu et al. (1973)
A quantitative definition
(poles in the complex plane)

\[ \sim \frac{i}{p^2 - m^2} \quad [\pi \text{ propagator}] \]
“poles correspond to particles”

\[ \sim iM \quad [\pi\pi \text{ scattering amplitude}] \]

poles:

- below $\pi\pi$-threshold \[\rightarrow \text{ bound state}\]
- above $\pi\pi$-threshold \[\rightarrow \text{ resonance} \rightarrow \text{ complex valued}\]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]

\[ -\frac{\Gamma}{2} = \text{Im}(E_R) / \text{MeV} \]

beautiful "bump"!
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]

\[
\begin{align*}
-\Gamma/2 &= \text{Im}(E_R) / \text{MeV} \\
|\mathcal{M}| &\quad E_{\text{cm}}/\text{MeV}
\end{align*}
\]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]
A toy example

\[ m_R = \frac{\text{Re}(E_R)}{\text{MeV}} \]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]
A toy example

\[ m_R = \text{Re}(E_R) / \text{MeV} \]

no “bump”, but there’s certainly a particle present
$$\mathcal{L}_{\text{QCD}} = \overline{\psi} (i \slashed{D} - m_f) \psi - \frac{1}{4} \text{tr} (G G)$$

“Seven parameters describe ALL of nuclear physics”

$$\Lambda_{\text{QCD}}, \frac{m_u}{\Lambda_{\text{QCD}}}, \frac{m_d}{\Lambda_{\text{QCD}}}, \frac{m_s}{\Lambda_{\text{QCD}}}, \frac{m_c}{\Lambda_{\text{QCD}}}, \frac{m_b}{\Lambda_{\text{QCD}}}, \alpha_{\text{QED}}$$

only dials at our disposal, no "tuning" can be done!
If only life were so easy!

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi} (i \mathcal{D} - m_f) \psi - \frac{1}{4} \text{tr} (G G) \]
\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_f (i \not{D} - m_f) \psi_f - \frac{1}{4} \text{tr} (GG) \]

Lattice QCD in four "easy" steps

1. **Discretize spacetime**

- A separation scale between all points in space.
- \(~0.1\text{fm}\)

more familiar lattices
\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_f (i \not{D} - m_f) \psi_f - \frac{1}{4} \text{tr} (G^2) \]

2. **Truncate spacetime**

- So it fits into a computer!
- ~4-6 fm

"It is a small world after all"
\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_f (i \slashed{D} - m_f) \psi_f - \frac{1}{4} \text{tr} (GG) \]

3. Tune quark masses

- Set them to physical values
- “Easier said than done!”
- Dial the quark masses
\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_f (i \mathcal{D} - m_f) \psi_f - \frac{1}{4} \text{tr} (GG) \]

4. Perform an actual calculation

Again, “Easier said than done!”
Finite vs. infinite volume spectrum

Infinite volume

Im[s]  Re[s]

bound state

thresholds

s = $E_{cm}^2$

narrow resonance

broad resonance
Finite vs. infinite volume spectrum

“only a finite number of modes can exist in a finite volume”
Finite vs. infinite volume spectrum

Infinite volume

\[ s = E_{\text{cm}}^2 \]

No resonance!

finite volume

scattering?
Scattering in finite volume: impossible!

Finite volume - a necessity for lattice QCD

- No asymptotic states, i.e., no scattering, resonances, etc.
- Challenging, but *not* an limitation
- Finite volume effects allow us to determine amplitudes

Huang & Yang (1957)
Lüscher (1986)
Lellouch & Lüscher (2000)
Physics in a 1D-box

\[ \phi(x) \sim e^{ipx} \]

Periodicity:

\[ L \ p_n = 2\pi n \]
Physics in a 1D-box

Two identical particles:

\[ \psi(x) \sim \cos(p |x| + \delta(p)) \]

Infinite volume scattering phase shift

Asymptotic wavefunction

Periodicity:

\[ L p_n + 2\delta(p_n) = 2\pi n \]
Physics in a 1D-box

\[ L \ p_n + 2\delta(p_n) = 2\pi n \]
Physics in a 1D-box

\[ L p_n + 2\delta(p_n) = 2\pi n \]
Physics in a 1D-box

\[ L \ p_n + 2\delta(p_n) = 2\pi n \]
Physics in a 1D-box

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\[ L \ p_n + 2\delta(p_n) = 2\pi n \]
Physics in a 1D-box

\[ L \ p_n + 2\delta(p_n) = 2\pi n \]
3D Quantization condition

Lüscher formalism

- Rummukainen and Gottlieb (1995)
- Bedaque (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and Sharpe (2005)

\[
\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0
\]

the most general two-particle quantization condition in a finite volume [RB (2014)]
hold for any number of two-particle states with any spin!
3D Quantization condition

\[ \det \left[ F^{-1}(P, L) + \mathcal{M}(P) \right] = 0 \]

the most general two-particle quantization condition in a finite volume \( \text{[RB (2014)]} \)
hold for any number of two-particle states with any spin!
Resonances in hadronic scattering
Extracting the spectrum

Two-point correlation functions:

\[ C_{ab}^{2pt.} (t, P) \equiv \langle 0 | O_b(t, P) O_a^\dagger (0, -P) | 0 \rangle \]
Extracting the spectrum

Two-point correlation functions:

\[ C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle \]

\[ = \sum_n \langle 0 | \mathcal{O}_b(t, \mathbf{P}) | n, L \rangle \langle n, L | \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle \]

insert complete set of states
Extracting the spectrum

Two-point correlation functions:

\[ C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle \]

\[ = \sum_n \langle 0 | \mathcal{O}_b(t, \mathbf{P}) | n, L \rangle \langle n, L | \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle \]

\[ = \sum_n \langle 0 | e^{t\hat{H}_{QCD}} \mathcal{O}_b(0, \mathbf{P}) e^{-t\hat{H}_{QCD}} | n, L \rangle \langle n, L | \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle \]

Remember Heisenberg operators in Euclidean spacetime?
Extracting the spectrum

Two-point correlation functions:

\[ C_{ab}^{2pt.}(t, P) \equiv \langle 0 | \mathcal{O}_b(t, P) \mathcal{O}_a^\dagger(0, -P) | 0 \rangle \]

\[ = \sum_n \langle 0 | \mathcal{O}_b(t, P) | n, L \rangle \langle n, L | \mathcal{O}_a^\dagger(0, -P) | 0 \rangle \]

\[ = \sum_n \langle 0 | e^{t \hat{H}_{QCD}} \mathcal{O}_b(0, P) e^{-t \hat{H}_{QCD}} | n, L \rangle \langle n, L | \mathcal{O}_a^\dagger(0, -P) | 0 \rangle \]

\[ = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t} \]

spectrum
\[ \text{det}\left[F^{-1}(P, L) + M(P)\right] = 0 \]

Wilson, RB, Dudek, Edwards & Thomas (2015)
Wilson, RB, Dudek, Edwards & Thomas (2015)
\( \pi\pi \) scattering
(spectrum and interpretation)

HadSpec Collaboration

\( \pi\pi \) in I=1 channel

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Comparing with experiment

cute, but aren’t experiments performed using $m_\pi=140$ MeV?

Quark-mass dependence of poles

Lin et al. (2009)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Quark-mass dependence of poles

- $\Gamma_\rho = 2 \text{Im}(E_\rho)/\text{MeV}$

$m_\rho = \text{Re}(E_\rho)/\text{MeV}$

$m_\pi = 391 \text{ MeV}$
$m_\pi = 536 \text{ MeV}$
$m_\pi = 700 \text{ MeV}$

$m_\pi = 236 \text{ MeV}$

$m_\pi = 140 \text{ MeV}$, Lattice QCD +χPT
$m_\pi = 140 \text{ MeV}$, Roy–Steiner

Lin et al. (2009)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Resonances in hadronic scattering
Resonances in electroweak processes
Sketch of formalism

1) Access matrix elements:

\[ C^{3pt.}_{2\rightarrow 1J} = \langle O_1(\delta t)J(t)O_2^\dagger(0) \rangle \rightarrow \langle 1|J|2 \rangle_L Z_1 Z_2^* e^{-(\delta t-t)E_1} e^{-tE_2} + \cdots \]
Sketch of formalism

1) Access matrix elements:

\[ C_{2\rightarrow 1J}^{3\text{pt.}} = \langle O_1(\delta t)J(t)O_2^\dagger(0) \rangle \rightarrow \langle 1|J|2 \rangle_L Z_1 Z_2^* e^{-(\delta t-t)E_1} e^{-tE_2} + \ldots \]

2) Interpret matrix elements:

\[ \left| \langle 2|J|1 \rangle_L \right|^2 = \mathcal{H} \mathcal{R} \mathcal{H} \]

RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)
Sketch of formalism

1) Access matrix elements:

$$C^{3pt.}_{2\rightarrow 1\mathcal{J}} = \langle O_1(\delta t)\mathcal{J}(t)O_2^\dagger(0) \rangle \rightarrow \langle 1|\mathcal{J}|2 \rangle_L Z_1 Z_2^* e^{-(\delta t-t)E_1} e^{-tE_2} + \cdots$$

2) Interpret matrix elements:

$$|\langle 2|\mathcal{J}|1 \rangle_L|^2 = \mathcal{H} \mathcal{R} \mathcal{H}$$

known finite volume function

$$\mathcal{R} \left( E_2, L, \delta, \frac{\partial \delta}{\partial E_2} \right)$$

RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)
Sketch of formalism

1) Access matrix elements:

\[ C_{2\rightarrow1J}^{3pt.} = \langle O_1(\delta t)J(t)O_2^\dagger(0) \rangle \longrightarrow \langle 1|J|2 \rangle_L Z_1 Z_2^* e^{-(\delta t-t)E_1} e^{-tE_2} + \cdots \]

2) Interpret matrix elements:

\[ \left| \langle 2|J|1 \rangle_L \right|^2 = \mathcal{H} \mathcal{R} \mathcal{H} \]
πγ*-to-ππ

Exploratory πγ*-to-ππ / πγ*-to-ϕ calculation:
- proof of principle/demonstration
- $m_{π} = 391$ MeV

HadSpec Collaboration

Wilson  Shultz  Thomas  Dudek  Edwards
πγ*-to-ππ amplitude

elastic ππ amplitude as function of energy [q-channel]
$\pi\gamma^*$-to-$\pi\pi$ amplitude

- elastic $\pi\pi$ amplitude as function of energy [$\rho$-channel]

- how does the $\pi\gamma^*$-to-$\pi\pi$ look like?

Graph showing the elastic $\pi\pi$ amplitude as a function of energy $E_{cm}/m_\pi$ with a peak at around $E_{cm}/m_\pi = 2.2$. The mass $m_\pi = 391$ MeV is also indicated.
**πγ*-to-ππ amplitude**

Lorentz invariant piece of:
\[
\langle \text{out}; \pi, P_\pi | J_{x=0}^\mu | \text{in}; \pi\pi, P_{\pi\pi}, \ell = 1 \rangle
\]

| $m_\pi \cdot |A_{\pi\pi, \pi\gamma}|$ |
|------------------|
| 0               |
| 1               |
| 2               |
| 3               |
| 4               |
| 5               |
| 6               |

<table>
<thead>
<tr>
<th>$E_{\text{cm}}/m_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>50</td>
</tr>
<tr>
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<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
</tbody>
</table>

- $Q^2 = 0$
- $Q^2 = 0.803 \text{ GeV}^2$
Intuitive explanation

The elastic $\pi\pi$ amplitude is dynamically enhanced by the presence of the $\rho$-meson.

Similarly, the $\pi\gamma^*$-to-$\pi\pi$ amplitude is enhanced by the $\omega$-meson.
Form factor at ϒ pole

Near the ϒ-pole, the $\pi\gamma^*$-to-$\pi\pi$ diverges

The residue is the $\pi\gamma^*$-to-$\Upsilon$ form factor

$\left(\right) \times \infty$
Form factor at $q$ pole

Shultz, Dudek, & Edwards (2014)

evaluated at the $q$-meson pole, $(853(2)-i 12.4(6)/2)$ MeV
Resonances in electroweak processes
The immediate future

Scalars: $\sigma/f_0(500), f_0(980), \ldots$

Wilson  Dudek  Edwards
The immediate future

Scalars: $\sigma/f_0(500), f_0(980), \ldots$

Resonances that decay to three-particles or more?

Hansen  Sharpe

Hansen  Sharpe
Collaborators

formalism
- Hansen
- Sharpe
- Walker-Loud

numerical
- Wilson
- Shultz
- Thomas
- Bolton

HadSpec Collaboration