Explore Hadron Structure
by
JLab12 Program &
Lattice QCD Calculations

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Lunch Seminar
21st Century Nuclear Science

- What is the role of QCD in the evolution of the universe?
- How hadrons are emerged from quarks and gluons?
- How does QCD make up the properties of hadrons?
  Their mass, spin, magnetic moment, ...
- What is the QCD landscape of nucleon and nuclei?
- Color Confinement  |  Asymptotic freedom
  |  200 MeV (1 fm)  |  2 GeV (1/10 fm)  |  Probing momentum
- How do the nuclear force arise from QCD?
- ...
New particles, new ideas, and new theories

- Early proliferation of new hadrons – “particle explosion”:

... and many more!
Hadrons have internal structure!

- Nucleons cannot be point-like spin-1/2 Dirac particles:

1933: Proton’s magnetic moment

\[ \mu_p = g_p \left( \frac{e\hbar}{2m_p} \right) \]
\[ g_p = 2.792847356(23) \neq 2! \]
\[ \mu_n = -1.913 \left( \frac{e\hbar}{2m_p} \right) \neq 0! \]

1960: Elastic e-p scattering

Form factors

Proton EM charge radius!

Electric charge distribution
New particles, new ideas, and new theories

- Early proliferation of new particles – “particle explosion”:

  ![Diagram showing the timeline of particle discoveries from 1890 to 1960.](image)

  - 1890-1900: Electron (e^-)
  - 1920-1930: Proton (p), Neutron (n), Positron (e^+), Muon (μ^±)
  - 1950-1960: η, η', ηc, ηb, η', ηc, ηb

  ... and many more!

- Quark Model

- Nobel Prize, 1969

- Proton: uud

- Neutron: udd
Deep inelastic scattering (DIS)

Modern Rutherford experiment – DIS (SLAC 1968)

- Localized probe:
  \[ Q^2 = -(p - p')^2 \gg 1 \text{ fm}^{-2} \]
  \[ \frac{1}{Q} \ll 1 \text{ fm} \]

- Two variables:
  \[ Q^2 = 4EE' \sin^2(\theta/2) \]
  \[ x_B = \frac{Q^2}{2m_N \nu} \]
  \[ \nu = E - E' \]

Discovery of spin \( \frac{1}{2} \) quarks, and partonic structure!

The birth of QCD (1973)
- Quark Model + Yang-Mill gauge theory

Nobel Prize, 1990
Quantum Chromo-dynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:**
  - **Quark fields:** spin-½ Dirac fermion (like electron)
  - **Color triplet:** \( i = 1, 2, 3 = N_c \)
  - **Flavor:** \( f = u, d, s, c, b, t \)

- **Gluon fields:** spin-1 vector field (like photon)
  - **Color octet:** \( a = 1, 2, ..., 8 = N_c^2 - 1 \)

- **QCD Lagrangian density:**
  \[
  \mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[ (i \partial_\mu \delta_{ij} - g A_{\mu,a}(t_a)_{ij}) \gamma^\mu - m_f \delta_{ij} \right] \psi_j^f \\
  - \frac{1}{4} \left[ \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2 \\
  + \text{gauge fixing + ghost terms}
  \]

- **QED – force to hold atoms together:**
  \[
  \mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}_i^f \left[ (i \partial_\mu - e A_\mu) \gamma^\mu - m_f \right] \psi_j^f - \frac{1}{4} \left[ \partial_\mu A_{\nu} - \partial_\nu A_\mu \right]^2
  \]

- **QCD Color confinement:**
  
  Gluons are dark, No free quarks or gluons ever been detected!
QCD Asymptotic Freedom

Interaction strength: \[ \alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi} \alpha_s(\mu_1) \ln \left( \frac{\mu_2^2}{\mu_1^2} \right)} \equiv \frac{4\pi}{-\beta_1 \ln \left( \frac{\mu_2^2}{\Lambda_{QCD}^2} \right)} \]

\( \mu_2 \) and \( \mu_1 \) not independent

Asymptotic Freedom \( \Leftrightarrow \) antiscreening

QCD: \[ \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0 \]

QED: \[ \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0 \]


Discovery of QCD
Asymptotic Freedom

Collider phenomenology
– Controllable perturbative QCD calculations

Nobel Prize, 2004
Our understanding of the nucleon evolves

Nucleon is a strongly interacting, relativistic bound state of quarks and gluons

QCD bound states:
- Neither quarks nor gluons appear in isolation!
- Understanding such systems completely is still beyond the capability of the best minds in the world

The great intellectual challenge:
Probe nucleon structure without “seeing” quarks and gluons?
How to test QCD dynamics?

We need the probe!

How to connect QCD quarks and gluons to observed hadrons and leptons?

QCD factorization and Universal distributions and correlations of quarks and gluons
Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons

- Large momentum transfer – sensitive to partons:

  Sensitive to partonic dynamics

  (Diagrams with more active partons from each hadron!)

- QCD factorization:

  Isolate QCD calculable short-distance partonic dynamics from hadronic matrix elements of parton fields – the structure
Hard probe and QCD factorization

- **One hadron:**

  \[
  \sigma_{\text{tot}}^{\text{DIS}} : \quad e^- e^- \rightarrow q \quad xP + O\left(\frac{1}{QR}\right)
  \]

  Hard-part Probe

  Parton-distribution Structure

  Power corrections Approximation

- **Two hadrons:**

  \[
  \sigma_{\text{tot}}^{\text{DY}} : \quad p + p \rightarrow \ldots + O\left(\frac{1}{QR}\right)
  \]

Predictive power:
Universal Parton Distributions
### Factorization for more than two hadrons

**Factorization for high $p_T$ single hadron:***

\[
\frac{d\sigma_{AB\rightarrow C+X}(p_A, p_B, p)}{dy dp_T^2} = \sum_{a,b,c} \phi_{A\rightarrow a}(x,\mu_F^2) \otimes \phi_{B\rightarrow b}(x',\mu_F^2)
\]

\[
\otimes \frac{d\hat{\sigma}_{ab\rightarrow c+X}(x,x',z,y, p_T^2\mu_F^2)}{dy dp_T^2} \otimes D_{c\rightarrow c}(z,\mu_F^2)
\]

**Fragmentation function:**

\[
D_{c\rightarrow c}(z,\mu_F^2)
\]

**Choice of the scales:**

\[
\mu_{Fac}^2 \approx \mu_{ren}^2 \approx p_T^2
\]

To minimize the size of logs in the coefficient functions

\[
\gamma, W/Z, \ell(s), \text{jet}(s), B, D, \Upsilon, J/\psi, \pi, ...
\]

\[
+ \mathcal{O}(1/P_T^2)
\]

\[
p_T \gg m \gtrsim \Lambda_{QCD}
\]
Hadron properties in QCD

How to understand the emergent hadron properties, such as the mass, spin, ..., from QCD dynamics of quarks and gluons?

Approximated model calculations, sum rules, and lattice calculations
Hadron properties in QCD

- **Mass** – intrinsic to a particle:
  - = Energy of the particle when it is at the rest
  - **QCD energy-momentum tensor in terms of quarks and gluons**
    \[ T^\mu{}^\nu = \frac{1}{2} \bar{\psi} i \mathcal{D}^{(\mu} \gamma^{\nu)} \psi + \frac{1}{4} g^\mu{}^\nu F^2 - F^\mu{}^\alpha F^{\nu}{}^\alpha \]
  - **Proton mass**:
    \[ m = \left. \frac{\langle p \mid \int d^3 x T^{00} \mid p \rangle}{\langle p \mid p \rangle} \right|_{\text{Rest frame}} \sim \text{GeV} \quad \text{X. Ji, PRL (1995)} \]

- **Spin** – intrinsic to a particle:
  - = Angular momentum of the particle when it is at the rest
  - **QCD angular momentum density in terms of energy-momentum tensor**
    \[ M^{\alpha\mu\nu} = T^{\alpha\nu} x^\mu - T^{\alpha\mu} x^\nu \]
  - **Proton spin**:
    \[ S(\mu) = \sum \langle P, S \mid \hat{J}_f^z (\mu) \mid P, S \rangle = \frac{1}{2} \]

*We do NOT know the state, \( |P, S\rangle \), in terms of quarks and gluons!*
A major success of QCD – is the right theory for the Strong Force!

How does QCD generate this? The role of quarks vs that of gluons?
Role of quarks and gluons – sum rules:

\[ m^2 \propto \langle p | T^\alpha_\alpha | p \rangle \]

\[ T^\alpha_\alpha = \frac{\beta(g)}{2g} F^{\mu\nu,a} F_{\mu\nu}^a + \sum_{q=u,d,s} m_q (1 + \gamma_m) \bar{\psi}_q \psi_q \]

\[ \beta(g) = -(11 - 2n_f/3) g^3/(4\pi)^2 + \ldots \]

At the chiral limit, the entire proton mass is from gluons!

Hadron mass in the rest frame – decomposition (sum rule):

\[ M = \frac{\langle P | H_{\text{QCD}} | P \rangle}{\langle P | P \rangle} \bigg|_{\text{rest frame}} = H_q + H_m + H_g + H_a \]

where

\[ H_q = \sum_q \bar{\psi}_q (-iD \cdot \alpha) \psi_q \quad \text{– quark energy} \]

\[ H_m = \sum_q \bar{\psi}_q m_q \psi_q \quad \text{– quark mass} \]

\[ H_g = \frac{1}{2} \left( E^2 + B^2 \right) \quad \text{– gluon energy} \]

\[ H_a = \frac{9 \alpha_s}{16\pi} (E^2 - B^2) \quad \text{– trace anomaly} \]
Hadron properties in QCD

_role of quarks and gluons – sum rules:_

◊ Partonic angular momenta, the contributions to hadron spin:

\[
S(\mu) = \sum_f \langle P, S|\hat{J}_f^z(\mu)|P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)
\]

where

\[
\hat{J}_q = \int d^3x \left[ \psi_q^\dagger \gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D})) \psi_q \right] \quad \text{– quark angular mom.}
\]

\[
\hat{J}_g = \int d^3x \left[ \vec{x} \times (\vec{E} \times \vec{B}) \right] \quad \text{– gluon angular mom.}
\]

◊ Spin decomposition (sum rule) – an incomplete story:

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + (L_q + L_g)
\]

Proton Spin

Different from QM!

Quark helicity: \( \frac{1}{2} \int dx (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}) \sim 30\% \) – Best known

Quark helicity: \( \Delta G = \int dx \Delta g(x) \sim 20\% \text{(with RHIC data)} \) – Start to know

 Orbital Angular Momentum (OAM): Not uniquely defined – Little known

_Gauge field is tied together with the motion of fermion – \( D^\mu \)!_
Hadron properties in QCD

- **Role of quarks and gluons – sum rules:**
  - Jaffe-Manohar’s quark OAM density:
    \[ \mathcal{L}_q^3 = \bar{\psi}_q \left[ \bar{x} \times (-i \bar{\partial}) \right]^3 \psi_q \]
  - Ji’s quark OAM density:
    \[ L_q^3 = \bar{\psi}_q \left[ \bar{x} \times (-i \bar{D}) \right]^3 \psi_q \]
  - **Difference** – generated by a “torque” of color Lorentz force
    \[ \mathcal{L}_q^3 - L_q^3 \propto \int \frac{d^2 y_T}{(2\pi)^3} \langle P' | \bar{\psi}_q(0) \gamma^+ \int_0^\infty d\zeta^- \Phi(0, \zeta^-) \Phi(\zeta, y) \psi(y) | P \rangle_{y^+ = 0} \sum_{i,j=1,2} [\epsilon^{3ij} y_T F^{ij} (\zeta^-)] \]

  “Chromodynamic torque”

*Sum rules are NOT unique – None of these matrix elements are physical!!!*

- **Value of the decomposition – a good sum rule:**
  
  *Every term of the sum rule is “independently measurable” – can be related to a physical observable with controllable approximation!*
How to understand the emergent hadron structures, such as the confined motion, spatial distributions, and correlations of quarks and gluons, ..., from QCD dynamics?

Measurements of PDFs, TMDs, GPDs, quark-gluon correlations, and Physics behind these functions?
Hadron structure in QCD

What do we need to know for the structure?

✧ In theory: \[ \langle P, S | \mathcal{O}(\bar{\psi}, \psi, A^\mu) | P, S \rangle \] – Hadronic matrix elements with all possible operators: \( \mathcal{O}(\bar{\psi}, \psi, A^\mu) \)

✧ In fact: None of these matrix elements is a direct physical observable in QCD – color confinement!

✧ In practice: Accessible hadron structure

1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
2) can be calculated in lattice QCD

Single-parton structure “seen” by a short-distance probe:

✧ 5D structure:

1) \( \int d^2 b_T \rightarrow f(x, k_T, \mu) \) – TMDs: 2D confined motion!

2) \( \int d^2 k_T \rightarrow F(x, b_T, \mu) \) – GPDs: 2D spatial imaging!

3) \( \int d^2 k_T d^2 b_T \rightarrow f(x, \mu) \) – PDFs: Number density!
Hadron structure in QCD

What do we need to know for the structure?

✧ In theory: \[ \langle P, S | O(\bar{\psi}, \psi, A^\mu) | P, S \rangle \] – Hadronic matrix elements with all possible operators: \[ O(\bar{\psi}, \psi, A^\mu) \]

✧ In fact: None of these matrix elements is a direct physical observable in QCD – color confinement!

✧ In practice: Accessible hadron structure = hadron matrix elements of quarks and gluons, which

1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or

2) can be calculated in lattice QCD

Multi-parton correlations:

\[ \sigma(Q, \bar{s}) \propto \left| \begin{array}{c} p, \bar{s} \rightarrow k_1 + k_2 + \cdots \end{array} \right| 2 \left( \frac{\langle k_\perp \rangle}{Q} \right)^n \] – Expansion

Quantum interference → 3-parton matrix element – not a probability!
Global QCD analyses of PDFs, ...

- Factorization for observables with identified hadrons:
  - One-hadron (DIS):
    \[ F_2(x_B, Q^2) = \sum_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2) \]
  - Two-hadrons (DY, Jets, W/Z, ...):
    \[ \frac{d\sigma}{dy dp_T^2} = \sum_f f'(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x') \]
  - DGLAP Evolution:
    \[ \frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \sum_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2) \]

- Input for QCD Global analysis/fitting:
  - World data with “Q” > 2 GeV
  - PDFs at an input scale:
    \[ \phi_{f/h}(x, \mu_0^2, \{\alpha_j\}) \]
Global QCD analysis – the machinery

Input PDFs at $Q_0$

$$\varphi_{f/h}(x, \{a_j\})$$

DGLAP

$Q^2$ – Evolution

$\varphi_{f/h}(x)$ at $Q>Q_0$

QCD calculation of cross sections

Minimize $\text{Chi}^2$

Vary $\{a_j\}$

Comparison with Data at various $x$ and $Q$

Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs
Global QCD analyses – a successful story

- Modern sets of PDFs @ NNLO:

Universal PDFs

NPDF2.3 (NNLO)

$xf(x, \mu^2 = 10 \text{ GeV}^2)$

$xf(x, \mu^2 = 10^4 \text{ GeV}^2)$

RHIC

STAR, PRL 97 (2006), 252001

- Combined MB
- Combined HT
- NLO QCD (Vogelsang)

CMS preliminary, 60 nb$^{-1}$

$\sqrt{s} = 7 \text{ TeV}$

- $|y| < 0.5 \times (1024)$
- $0.5 \leq |y| < 1.0 \times (256)$
- $1.0 \leq |y| < 1.5 \times (64)$
- $1.5 \leq |y| < 2.0 \times (16)$
- $2.0 \leq |y| < 2.5 \times (4)$
- $2.5 \leq |y| < 3.0 \times (1)$

LHC

NLO pQCD+NP

Exp. uncertainty

Anti-$K_T$, $R=0.5$ PF

Tevatron

$\frac{d^2\sigma}{dy dp_T}$ (pb/GeV)
Uncertainties of PDFs

“non-singlet” sector

“singlet” sector
Partonic luminosities

**q - qbar**

NNLO $\Sigma_q(q\bar{q})$ luminosity at LHC ($\sqrt{s} = 8$ TeV)

- Ratio to MSTW 2008 (68% C.L.)
- $M_W, M_Z$

**g - g**

NNLO $gg$ luminosity at LHC ($\sqrt{s} = 8$ TeV)

- Ratio to MSTW 2008 (68% C.L.)
- $M_{Higgs}, 2m_{top}$
PDFs at large $x$

- **Testing ground for hadron structure at $x \to 1$:**
  - $d/u \to 1/2$
  - $d/u \to 0$
  - $d/u \to 1/5$
  - $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$

  - **SU(6) Spin-flavor symmetry**
  - **Scalar diquark dominance**
  - **pQCD power counting**
  - **Local quark-hadron duality**

Impact of JLab 6 GeV data

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**PRD 93 (2016) 074005 & 114017**
PDFs at large $x$

- **Testing ground for hadron structure at** $x \to 1$:
  - $d/u \to 1/2$
    - SU(6) Spin-flavor symmetry
  - $d/u \to 0$
    - Scalar diquark dominance
  - $d/u \to 1/5$
    - pQCD power counting
  - $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2}$
    - Local quark-hadron duality
    - $\approx 0.42$
  - $\Delta u/u \to 2/3$
    - $\Delta d/d \to -1/3$
  - $\Delta u/u \to 1$
    - $\Delta d/d \to -1/3$

Both JLab12 and lattice QCD can help!
Upcoming experiments – JLab12

- NSAC milestone HP14 (2018):
  - MARATHON
  - BONUS
  - CLAS12

  \[ \frac{F_{2n}}{F_{2p}} \]

  Systematic error due to experimental and theoretical uncertainties from point-to-point

  \[ d/u = 1/5 \]

  \[ d/u = 0 \]

  Plus many more JLab experiments:

  - E12-06-110 (Hall C on $^3$He),
  - E12-06-122 (Hall A on $^3$He),
  - E12-06-109 (CLAS on NH$_3$, ND$_3$), …

  and Fermilab E906, …

  Plus complementary Lattice QCD effort
Lattice calculations of hadron structure

Lattice QCD

New ideas – from quasi-PDFs (lattice calculable) to PDFs:

✧ High $P_z$ effective field theory approach:

$$\tilde{q}(x, \mu^2, P_z) = \int_x^1 \frac{dy}{y} Z \left( \frac{x}{y}, \frac{\mu}{P_z} \right) q(y, \mu^2) + \mathcal{O} \left( \frac{\Lambda^2}{P_z^2}, \frac{M^2}{P_z^2} \right)$$

✧ QCD collinear factorization approach:

$$\tilde{q}(x, \mu^2, P_z) = \sum_f \int_0^1 \frac{dy}{y} C_f \left( \frac{x}{y}, \frac{\mu^2}{\bar{\mu}^2}, P_z \right) f(y, \bar{\mu}^2) + \mathcal{O} \left( \frac{1}{\mu^2} \right)$$

Non-perturbative lattice UV renormalization:

Effective mass renormalization, Gradient flow, …

The TMD Collaboration + on-going effort around the world!

Plus the intense local JLab theory effort!
Lattice calculations of hadron structure

Lattice QCD

- New ideas – from quasi-PDFs (lattice calculable) to PDFs:
  - High $P_z$ effective field theory approach:
    $$\bar{q}(x, \mu^2, P_z) = \int_x^1 \frac{dy}{y} Z \left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y, \mu^2) + O \left(\frac{\Lambda^2}{P^2_z}, \frac{M^2}{P^2_z}\right)$$
  - QCD collinear factorization approach:
    $$\bar{q}(x, \mu^2, P_z) = \sum_f \int_0^1 \frac{dy}{y} C_f \left(\frac{x}{y}, \frac{\mu^2}{\bar{\mu}^2}, P_z\right) f(y, \bar{\mu}^2) + O \left(\frac{1}{\mu^2}\right)$$

Non-perturbative lattice UV renormalization:
  Effective mass renormalization, Gradient flow, ...

- Tremendous potentials!

  - PDFs of proton, neutron, pion, …; TMDs, GPDs, …; JLab12 expts
How does the gluon distribution saturate at small $x$? Which “glue” the quarks together. But experiments probing proton structure at the HERA collider at Germany’s DESY laboratory, and the increasing body of evidence from RHIC and LHC, suggest that this picture is far too simple. Countless other gluons and a “sea” of quarks and anti-quarks pop in and out of existence within each hadron. These fluctuations can be probed in high energy scattering experiments: due to Lorentz time dilation, the more we accelerate a proton and the closer it gets to the speed of light, the longer are the lifetimes of the gluons that arise from the quantum fluctuations. An outside “observer” viewing a fast moving proton would see the cascading of gluon s as long as longer the larger the velocity of the proton. So, in effect, by speeding up the proton, one can slow down the gluon fluctuations enough to “take snapshots” of the mw with particles enough to interact with the high-energy proton.

In DIS experiments one probes the proton wave function with a lepton, which interacts with the proton by exchanging a (virtual) photon with it (see the Sidebar on page ... ). The virtuality of the photon $Q^2$ determines the size of the region in the plane transverse to the beam axis probed by the photon: by uncertainty principle the region’s width is $\Delta r_\perp \sim 1/Q^2$. Another relevant variable is Bjorken $x$, which is the fraction of the proton momentum carried by the struck quark. At high energy $x \approx Q^2/W^2$ is small ($W^2$ is the center-of-mass energy squared of the photon-proton system): therefore, small $x$ corresponds to high energy scattering.
Two-momentum-scale observables

- Cross sections with two-momentum scales observed:
  \[ Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{QCD} \]
  - Hard scale: \( Q_1 \) localizes the probe to see the quark or gluon d.o.f.
  - "Soft" scale: \( Q_2 \) could be more sensitive to hadron structure, e.g., confined motion

- Two-scale observables with the hadron broken:
  - SIDIS: \( Q \gg P_T \)
  - DY: \( Q \gg P_T \)
  - Two-jet momentum imbalance in SIDIS, ...
  - Natural observables with TWO very different scales
  - TMD factorization: partons’ confined motion is encoded into TMDs
Two-momentum-scale observables

- Cross sections with two-momentum scales observed:
  \[ Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{QCD} \]
  - **Hard scale**: \( Q_1 \) localizes the probe to see the quark or gluon d.o.f.
  - **“Soft” scale**: \( Q_2 \) could be more sensitive to hadron structure, e.g., confined motion

- Two-scale observables with the hadron unbroken:
  - DVCS: \( Q^2 \gg |t| \)
  - DVEM: \( Q^2 \gg |t| \)
  - EHMP: \( Q^2 \gg |t| \)
  - Natural observables with TWO very different scales
  - GPDs: Fourier Transform of t-dependence gives spatial b\(_T\)-dependence

\[ t = (p_1 - p_2)^2 \]
Lepton-Hadron Facility: from JLab to EIC

- Precision of lepton probe + energy of hadron machine:

\[ Q^2 \rightarrow \text{Measure of resolution} \]
\[ y \rightarrow \text{Measure of inelasticity} \]
\[ x \rightarrow \text{Measure of momentum fraction of the struck quark in a proton} \]
\[ Q^2 = S \times y \]

- Inclusive events: \( e+p/A \rightarrow e'+X \)
  Detect only the scattered lepton in the detector

- Semi-Inclusive events: \( e+p/A \rightarrow e'+h(\pi,K,p,jet)+X \)
  Detect the scattered lepton in coincidence with identified hadrons/jets

- Exclusive events: \( e+p/A \rightarrow e'+p'/A'+h(\pi,K,p,jet) \)
  Detect every things including scattered proton/nucleus (or its fragments)
Semi-inclusive DIS (SIDIS)

Process:

$$e(k) + N(p) \rightarrow e'(k') + h(P_h) + X$$

Natural event structure:

In the photon-hadron frame: $$P_{hT} \approx 0$$

Semi-Inclusive DIS is a natural observable with TWO very different scales

$$Q \gg P_{hT} \gtrsim \Lambda_{QCD}$$

Localized probe sensitive to parton’s transverse motion

Soft gluons are everywhere:

TMD fragmentation

Soft factors

TMD parton distribution
Semi-inclusive DIS (SIDIS)

- **Process:**
  \[ e(k) + N(p) \rightarrow e'(k') + h(P_h) + X \]

- **Natural event structure:**
  In the photon-hadron frame: \( P_{hT} \approx 0 \)
  
  Semi-Inclusive DIS is a natural observable with TWO very different scales
  
  \[ Q \gg P_{hT} \gtrsim \Lambda_{QCD} \]
  Localized probe sensitive to parton’s transverse motion

- **QCD factorization - approximation:**
  ✧ Low \( P_{hT} \) – TMD factorization:
  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes D_{f\rightarrow h}(z, p_\perp) \otimes S(k_{s\perp}) + \mathcal{O}\left(\frac{P_{h\perp}}{Q}\right)
  \]

  ✧ High \( P_{hT} \) – Collinear factorization:
  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\rightarrow h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)
  \]

  ✧ \( P_{hT} \) Integrated - Collinear factorization:
  \[
  \sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\rightarrow h} + \mathcal{O}\left(\frac{1}{Q}\right)
  \]
SIDIS is the best for probing TMDs

- Naturally, two scales & two planes:

\[ A_{UT}(\varphi_h, \varphi_S) = \frac{1}{P} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \]

\[ = A_{UT}^{Collins} \sin(\phi_h + \phi_S) + A_{UT}^{Sivers} \sin(\phi_h - \phi_S) + A_{UT}^{Pretzelosity} \sin(3\phi_h - \phi_S) \]

- Separation of TMDs:

\[ A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp \]

\[ A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1 \]

\[ A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp \]

**Hard, if not impossible, to separate TMDs in hadronic collisions**

Using a combination of different observables (not the same observable):
- jet, identified hadron, photon, ...
Parton $k_T$ at the hard collision

- Sources of parton $k_T$ at the hard collision:
  - Gluon shower
  - Confined motion
  - Emergence of a hadron hadronization

- Large $k_T$ generated by the shower (caused by the collision):
  - $Q^2$-dependence – linear evolution equation of TMDs in $b$-space
  - The evolution kernels are perturbative at small $b$, but, not large $b$
  - The nonperturbative inputs at large $b$ could impact TMDs at all $Q^2$

- Challenge: to extract the “true” parton’s confined motion:
  - Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs
Different fits – different Q-dependence

- Aybat, Prokudin, Rogers, 2012:
  - Huge Q dependence

- Sun, Yuan, 2013:
  - Smaller Q dependence

No disagreement on evolution equations!

Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent “form factor”
“Predictions” for $A_N$ of W-production at RHIC?

- **Sivers Effect:**
  - Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons.
  - QCD Prediction: Sign change of Sivers function from SIDIS and DY.

- **Current “prediction” and uncertainty of QCD evolution:**

  ![Graphs showing $A_N$ vs. $y$ for W-production at RHIC.]

  TMD collaboration proposal: Lattice, theory & Phenomenology.

  RHIC is the excellent and unique facility to test this (W/Z – DY)!
Unified view of nucleon structure

- Wigner distributions:
  - Momentum Space
  - Coordinate Space
  - TMDs
  - GPDs
  - Confined motion
  - Spatial distribution
  - Two-scales observables

- Sivers Functions

- Position $\mathbf{r} \times$ Momentum $\mathbf{p} \rightarrow$ Orbital Motion of Partons

\[
\int d^2b_T f(x, b_T) \quad \int d^2k_T f(x, k_T)
\]
Orbital angular momentum

OAM: Correlation between parton’s position and its motion – in an averaged (or probability) sense

- Jaffe-Manohar’s quark OAM density:
  \[ L^3_q = \bar{\psi}_q \left( \vec{x} \times (-i \vec{\partial}) \right)^3 \psi_q \]

- Ji’s quark OAM density:
  \[ L^3_q = \bar{\psi}_q \left( \vec{x} \times (-i \vec{D}) \right)^3 \psi_q \]

- Difference between them:
  - compensated by difference between gluon OAM density
  - represented by different choice of gauge link for OAM Wagner distribution

\[
L^3_q \{ L^3_q \} = \int dx \, d^2 b \, d^2 k_T \left( \vec{b} \times \vec{k}_T \right)^3 \mathcal{W}_q(x, \vec{b}, \vec{k}_T) \left\{ W_q(x, \vec{b}, \vec{k}_T) \right\}
\]

with

\[
\mathcal{W}_q \{ W_q \} (x, \vec{b}, \vec{k}_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i \vec{\Delta}_T \cdot \vec{b}} \int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{i(x P^+ y^- - \vec{k}_T \cdot \vec{y}_T)}
\]

JM: “staple” gauge link

Ji: straight gauge link

between 0 and \( y=(y^+=0, y^-, y_T) \)

Hatta, Lorce, Pasquini, …
Orbital angular momentum

OAM: Correlation between parton’s position and its motion – in an averaged (or probability) sense

- Jaffe-Manohar’s quark OAM density:
  \[ L_q^3 = \psi_q^\dagger \left[ \vec{x} \times (-i \vec{\partial}) \right]^3 \psi_q \]

- Ji’s quark OAM density:
  \[ L_q^3 = \psi_q^\dagger \left[ \vec{x} \times (-i \vec{D}) \right]^3 \psi_q \]

- Difference between them:
  
  - generated by a “torque” of color Lorentz force

  \[ L_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P' | \overline{\psi}_q(0) \gamma^+ \int_{y^-}^{\infty} dz^- \Phi(0, z^-) \times \sum_{i,j=1,2} [\epsilon^{3ij} y_T^i F^{+j}(z^-)] \Phi(z^-, y) \psi(y) | P \rangle \big|_{y^+ = 0} \]

  “Chromodynamic torque”

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of \( g_2 \)
Nucleon spin and OAM from lattice QCD

\[ \chi \text{QCD Collaboration:} \]

Nucleons have two types of interactions:

- **Connected Interaction (CI)**
  - Connected contributions to nucleon spin and OAM from lattice QCD
  - Pie chart showing contributions: $J^{u+d}$ (CI) 63.5%, $J^{u+d}$ (DI) 7%, $J^s$ (DI) 28.8%, $J^g$ 2.2%

- **Disconnected Interaction (DI)**
  - Disconnected contributions to nucleon spin and OAM from lattice QCD
  - Pie chart showing contributions: $L^{u+d}$ (CI) 25.12%, $L^{u+d}$ (DI) 14.1%, $L^s$ (DI) 32.2%, $J^g$ 1%

[Deka et al. arXiv:1312.4816]
Why 3D hadron structure?

Rutherford’s experiment – atomic structure (100 years ago):

Atom:

J.J. Thomson’s plum-pudding model

Rutherford’s planetary model

Modern model Quantum orbitals

Discovery of nucleus
A localized charge/force center
A vast “open” space

Discovery of Quantum Mechanics, and the Quantum World!

Completely changed our “view” of the visible world:

- Mass by “tiny” nuclei – less than 1 trillionth in volume of an atom
- Motion by quantum probability – the quantum world!
- Provided infinite opportunities to improve things around us, …

What would we learn from the hadron structure in QCD, …?
Why 3D nucleon structure?

- Spatial distributions of quarks and gluons:
  - **Bag Model:**
    - Gluon field distribution is wider than the fast moving quarks.
    - Gluon radius > Charge Radius
  - **Constituent Quark Model:**
    - Gluons and sea quarks hide inside massive quarks.
    - Gluon radius ~ Charge Radius
  - **Lattice Gauge theory (with slow moving quarks):**
    - Gluons more concentrated inside the quarks
    - Gluon radius < Charge Radius

3D Confined Motion (TMDs) + Spatial Distribution (GPDs)
Relation between charge radius, quark radius (x), and gluon radius (x)?
QCD has been extremely successful in interpreting and predicting high energy experimental data!

But, we still do not know much about hadron structure – work just started!

Cross sections with large momentum transfer(s) and identified hadron(s) are the source of structure information.

QCD factorization is necessary for any controllable “probe” for hadron’s quark-gluon structure!

JLab12/Lattice QCD calculations, plus the future EIC, will provide answers to many of our questions on hadron properties and structure, ...

Thank you!
BACKUP SLIDES
New particles, new ideas, and new theories

- Proliferation of new particles – “November Revolution”:
  - Quark Model
  - QCD
  - EW
  - Completion of SM?
New particles, new ideas, and new theories

- Proliferation of new particles – “November Revolution”:

  Quark Model
  QCD
  EW

  November Revolution!

  Completion of SM?

  A new particle explosion?
What holds hadron together – the glue?

- Understanding the glue that binds us all – the Next QCD Frontier!

- Gluons are weird particles!
  - Massless, yet, responsible for nearly all visible mass

"Mass without mass!"

Higgs mechanism

Quarks
Mass $\approx 1.78 \times 10^{-26}$ g

$\sim 1\%$ of proton mass

Dynamics of gluons

Proton
Mass $\approx 168 \times 10^{-26}$ g

$\sim 99\%$ of proton mass

Bhagwat & Tandy/Roberts et al
Gluons are weird particles!

- Massless, yet, responsible for nearly all visible mass
- Carry color charge, responsible for color confinement and strong force

Force between a heavy quark pair

Heavy quarks experience a force of \(~16\) tons at \(~1\) Fermi \((10^{-15}\text{ m})\) distance
What holds it together – the glue?

- Understanding the glue that binds us all – the Next QCD Frontier!

- Gluons are weird particles!
  - Massless, yet, responsible for nearly all visible mass
  - Carry color charge, responsible for color confinement and strong force
  - but, also for asymptotic freedom

\[ \frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)} = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi} \alpha_s(\mu_1) \ln \left( \frac{\mu_2^2}{\mu_1^2} \right)} = \frac{4\pi}{-\beta_1 \ln \left( \frac{\mu_2^2}{\Lambda_{\text{QCD}}^2} \right)} \]

QCD perturbation theory
Gluons are weird particles!

- Massless, yet, responsible for nearly all visible mass
- Carry color charge, responsible for color confinement and strong force but, also for asymptotic freedom, as well as the abundance of glue
Gluons are wired particles!

- Massless, yet, responsible for nearly all visible mass
- Carry color charge, responsible for color confinement and strong force
  but, also for asymptotic freedom, as well as the abundance of glue

**Without gluons, there would be NO nucleons, NO atomic nuclei…**

**NO visible world!**

Need an EIC!