

Probing Multiparton Correlations at CEBAF

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Abstract

In this talk, I explore the possibilities of probing the multiparton correlation functions at CEBAF at its current energy and the energies with its future upgrades.

I. INTRODUCTION

The perturbation theory of Quantum Chromodynamics (QCD) has been very successful in interpreting the data from high energy experiments. For observables involving hadrons, QCD factorization theorem enables us to systematically separate the short-distance partonic dynamics from the long-distance non-perturbative hadronic informations. In particular, when the energy exchange in a collision is high enough, the QCD factorization theorem factorizes all non-perturbative information into a set of universal parton distributions. Such universal parton distributions have been successfully extracted from the existing data.

However, the nonperturbative dynamics of the strong interactions is much richer than what we have learned through the parton distributions. The parton distributions are interpreted as the probability densities to find the partons within a hadron. Knowing only such probability densities is not enough to understand the full partonic dynamics within a hadron. But, very little has been learned about the multiparton correlations. This is because the physics associated with the multiparton correlations is directly related to multiple scattering, and most physical observables measured in existing high energy experiments are dominated by the contributions from the single hard scattering. And, in general, the multiple scattering is power suppressed in comparison with the single scattering. Therefore, it is often the case that even an excellent data cannot provide a good enough information for extracting the multiparton correlations, after removing the large contributions from the single scattering.

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In this talk, I use examples to show that (1) by taking the advantage of kinematics, we can enhance the contribution from the multiparton scattering; and (2) there are new types of physical observables which eliminate the contribution from the single scattering. The rest of this talk is organized as follows. A general discussion on relationships between the multiple scattering, the multiparton correlation functions and the high twist operators is given in next section. In Sec. III, with some simple assumptions, we show analytically that when x_B is large, the complete twist-4 contributions to the DIS structure functions can be simplified and is proportional to the derivative of the twist-2 parton distributions. In Sec. IV, we demonstrate how to extract multiparton correlation functions inside a large nucleus. Finally, in Sec. V, we provide our summary and discussion on why CEBAF is a good place to measure the multiparton correlation functions.

II. MULTIPARTON CORRELATIONS AND HIGH TWIST

Multiparton correlation functions are defined as the Fourier transform of the matrix elements of the multiparton fields,

$$T_m(k_1, k_2, \dots, k_{n-1}) \propto \int \prod_{m=1}^{n-1} \frac{d^4 y_m}{(2\pi)^4} e^{i k_m \cdot y_m} \times \langle N | \phi(0) \phi(y_1) \dots \phi(y_{n-1}) | N \rangle, \quad (1)$$

where $|N\rangle$ is the hadron state, the k_i with $i = 1, \dots, n - 1$ are independent parton momenta, and the ϕ 's are the parton field operators. Because of the momentum dependence of the correlation functions, the multiparton field operators in the position space, $\phi(0)\phi(y_1)\dots\phi(y_{n-1})$ are not local. In a gauge theory, like QCD, a path integral of the gauge field is often necessary to be sandwiched between the ϕ 's in Eq. (1), in order to make the correlation functions gauge invariant [1]. The simplest correlation functions are the matrix elements of operators with two parton fields. For example, the quark distribution of a proton of momentum p , moving in the “+” direction,

$$q(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p | \bar{\psi}_q(0) \left(\frac{\gamma^+}{2} \right) \psi_q(y^-) | p \rangle, \quad (2)$$

which is a correlation of two quark fields.

Beyond two-parton correlation functions, the choice of the multiparton field operators is not unique because of the spin structure of the quark fields, the equation of motion, and the gauge invariance. For example, the structure functions measured in deeply inelastic scattering (DIS) can have following factorized form,

$$F(x_B, Q^2) = \sum_m C^{(m)}(x_B, x_1, x_2, \dots, Q^2/\mu^2) \otimes T^{(m)}(x_1, x_2, \dots, \mu^2) \left(\frac{1}{Q^2}\right)^m, \quad (3)$$

where x_B is the Bjorken variable, and Q^2 is the large virtual momentum exchange in DIS. In Eq. (3), the coefficient functions, $C^{(m)}$'s, are calculable within the QCD perturbation theory, and the $T^{(m)}$'s are nonperturbative and defined as the Fourier transform of the matrix elements of the multiparton field operators. The term with $m = 0$ is often called the leading twist or leading power contribution, while the terms with $m \neq 0$ are called the high twist contributions or power corrections. It was pointed out in Ref. [2] that at a given order of power corrections, m , the $T^{(m)}$ can be expressed in terms of the multiparton correlation functions with the same number of physical parton fields (i.e., fields with physical polarizations). For example, the $T^{(1)}$ for the $m = 1$ power corrections in DIS can be expressed in terms of *four*-parton correlation functions only (note, the power corrections due to the hadron mass are not included in discussion here).

Although the *four*-parton correlation functions correspond to the *leading* power corrections to the structure functions in DIS or other physical observables, we have not had much success in extracting the information on these correlation functions. This is because of following obvious difficulties:

- The twist-4 (or higher twist) contributions are power suppressed in comparison with the corresponding leading twist contributions.
- There are many more four-parton correlation functions than the two-parton correlation functions (or parton distributions) due to the rich spin decompositions and flavor combinations of different parton field operators.
- Dependence on extra parton momenta requires much more information to extract multiparton correlation functions than the parton distributions.

In order to extract the multiparton correlation functions, we have to identify the physical observables such that above difficulties can be minimized. For example, we can measure a physical observable in a particular part of the phase space so that the power corrections are relatively enhanced; we can identify an observable which gets no leading twist contribution; we can take an advantage of kinematics and identify observables with a limited leading subprocesses so that only a small number of multiparton correlation functions are relevant [3, 4]. In the rest of this talk, we will give two examples to show how to extract the four-parton correlation functions, and provide the physical interpretation of these four-parton correlation functions.

III. TWIST-4 CONTRIBUTION TO THE STRUCTURE IN DEEPLY INELASTIC SCATTERING

In this section, we demonstrate how to extract the leading power corrections (i.e., the twist-4 contributions) to the structure functions in DIS at the CEBAF energies. In particular, we provide the physical interpretation of the leading power corrections. Note, the inclusion of target mass corrections are straightforward, and will not be discussed here.

Although the twist-4 contributions to the structure functions in DIS are power suppressed by a factor of $(1/Q^2)$, as shown in Eq. (3), the corresponding coefficient functions $C^{(1)}(x_B, x_1, x_2, \dots)$ are larger than the leading $C^{(0)}(x_B)$'s for large x_B . The structure functions at large x_B are useful for extracting the information on the size of power corrections [5]. However, after removing the leading twist contributions, the existing data are still not good enough for extracting detailed information on different four-parton correlation functions $T^{(1)}$'s in Eq. (3). It is a common practice to use the following parameterization for the leading power corrections [5],

$$F_2(x_B, Q^2) \approx \left(1 + \frac{h(x)}{Q^2}\right) F_2(x_B, Q^2)_{\text{LT}}, \quad (4)$$

where $F_2(x_B, Q^2)_{\text{LT}}$ includes the leading twist (or leading power) contribution and the target mass corrections to the full structure function $F_2(x_B, Q^2)$. The $h(x)$ in Eq. (4) is a fitting function. Yang et al. [5] used following parameterization,

$$h(x_B) = a \left(\frac{x_B^b}{1 - x_B} - c \right), \quad (5)$$

to fit the existing DIS data and extract the values of the parameters a, b and c for different targets. On the other hand, a complete expression of the twist-4 contributions to the DIS structure functions at the leading order of α_s were derived more than fifteen years ago [2, 6]. It is important to find the linkage between the phenomenological fits and the theoretical formulas, and reveal the physical meanings of these fitting parameters.

In terms of one photon exchange, the invariant mass square of the hadron-photon system is given by

$$W^2 = (p + q)^2 \approx \frac{Q^2}{x_B} (1 - x_B). \quad (6)$$

For inclusive DIS, it is obvious that the power expansion of the structure functions in Eq. (3) should have terms proportional to $(1/W^2)^m$. Because W^2 shown in Eq. (6) can be much smaller than Q^2 in large x_B region, the power corrections are relatively more important in this region. Without worrying about the target mass corrections, the coefficient functions should have following asymptotic behavior,

$$C^{(m)}(x_B, x_1, x_2, \dots, Q^2/\mu^2) \implies \left(\frac{1}{1 - x_B} \right)^m \quad (7)$$

as $x_B \rightarrow 1$. From above simple argument based on the DIS kinematics, it seems natural to parameterize the fitting function $h(x_B)$ in Eq. (5).

In addition to the asymptotic behavior of the coefficient functions, it is also important to understand why the twist-4 part of the structure function, $F_2(x_B, Q^2)_{\text{TW4}}$, which are proportional to the four-parton correlation functions, can be approximated to be proportional to the twist-2 part of the structure functions, as shown in Eq. (4). The twist-4 contributions to the structure functions at the leading order of α_s are given by [2, 6]

$$F_2(x_B, Q^2)_{\text{TW4}} = \frac{x_B}{Q^2} \sum_q e_q^2 \left[4T_{qD_1}^{(1)}(x_B) - x_B \int dx_1 dx_2 \times \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1} T_{qD_2}^{(1)}(x_2, x_1) \right]; \quad (8)$$

and

$$F_L(x_B, Q^2)_{\text{TW4}} = \frac{1}{Q^2} \sum_q e_q^2 \left[4T_{qD_1}^{(1)}(x_B) \right], \quad (9)$$

plus the contributions from the four-quark correlation functions. In Eqs. (8) and (9), the quark-gluon correlation functions are defined as

$$T_{qD_1}^{(1)}(x_B) = \frac{1}{4} \int \frac{dy^-}{2\pi} e^{ix_B p^+ y^-} \langle p | \bar{\psi}_q(0) \gamma_\alpha \gamma^+ \gamma_\beta D_T^\alpha(0) D_T^\beta(y^-) \psi_q(y^-) | p \rangle; \quad (10)$$

and

$$T_{qD_2}^{(1)}(x_B) = \frac{1}{4} \int \frac{dy^-}{2\pi} \frac{p^+ dy_1^-}{2\pi} e^{ix_1 p^+ y^-} e^{i(x_2 - x_1) p^+ y_1^-} \times \langle p | \bar{\psi}_q(0) \gamma_\alpha \gamma^+ \gamma_\beta D_T^\alpha(y_1^-) D_T^\beta(y_1^-) \psi_q(y_1^-) | p \rangle, \quad (11)$$

where D_T are transverse components of the covariant derivatives.

In Ref. [7], we show that when x_B is large, a simplified expression, similar to the phenomenological parameterization, can be derived from Eq. (8). Key arguments and assumption of the derivation are summarized as follows:

- It is argued that the correlation function $T_{qD_1}^{(1)}(x_B)$ cannot be proportional to $F_2(x_B, Q^2)_{LT}/(1 - x_B)$ as $x_B \rightarrow 1$. Therefore, the leading contribution to $F_2(x_B, Q^2)_{TW4}$ is from the second term in Eq. (8).
- Assuming that $D_T^2(y_1^-) \equiv -g_{\alpha\beta} D_T^\alpha(y_1^-) D_T^\beta(y_1^-)$ is a very slow function of y_1^- , we can reduce the quark-gluon correlation function $T_{qD_2}^{(1)}$ to the normal twist-2 quark distribution defined in Eq. (2).
- It is argued that the contributions from the four-quark correlation functions are smaller than that from the quark-gluon correlation functions given in Eq. (8).

With the above arguments and assumption, we derive

$$F_1(x_B, Q^2)_{TW4} = \frac{1}{2} \left[\frac{F_2(x_B, Q^2)_{TW4}}{x_B} - F_L(x_B, Q^2)_{TW4} \right] \approx \frac{1}{2} \sum_q e_q^2 \frac{\langle D_T^2 \rangle}{2Q^2} x_B \frac{d}{dx_B} q(x_B), \quad (12)$$

where the twist-2 quark distribution $q(x_B)$ is defined in Eq. (2). The $\langle D_T^2 \rangle$ is an averaged value of the covariant derivative. Eq. (12) is a direct result of

above arguments and assumption. As shown below, Eq. (12) can be reduced to an expression very similar the phenomenological parameterization given in Eqs. (4) and (5).

Eq. (12) in its present form is still different from the parameterization shown in Eq. (5) because of the derivative of the quark distribution. However, when x_B is large, the quark distribution should have following behavior

$$q(x_B) \implies (1 - x_B)^{\alpha_q} \quad \text{as } x_B \rightarrow 1, \quad (13)$$

with the real parameter $\alpha_q \sim 3.0$ for the valence quark distributions. Therefore, we have

$$x_B \frac{d}{dx_B} q(x_B) = \alpha_q \frac{x_B}{1 - x_B} q(x_B) + \text{terms without } \frac{1}{1 - x_B}. \quad (14)$$

Substituting Eq. (14) into Eq. (12), we obtain

$$F_2(x_B, Q^2)_{\text{TW4}} \approx \sum_q e_q^2 x_B q(x_B) \frac{\alpha_q \langle D_T^2 \rangle}{2Q^2} \frac{x_B}{1 - x_B} + \text{terms without } \frac{1}{1 - x_B}, \quad (15)$$

It is now clear that our analytical result shown in Eq. (12) or equivalently Eq. (15) is very similar to the phenomenological parameterization shown in Eqs. (4) and (5). The difference is that our result was derived analytically from the complete leading order twist-4 contributions with some simple arguments and assumptions, and thus, every parameter have the physical interpretations. In addition, because of the dependence on the derivative of the quark distributions, instead of the leading twist structure function, our result provides the natural difference between the proton target and the neutron target, which is observed in the NMC data.

In order to measure the power corrections to the structure functions, we like to keep x_B large and Q^2 small. However, from Eq. (6), we cannot keep Q^2 too small, because of the hadronic resonances when W is less than 2 GeV. When the invariant mass of the “hadron-photon” system is less than 2 GeV, the power expansion shown in Eq. (3) fails. Therefore, to test the perturbative power expansion of the structure functions and to measure

the multiparton correlation functions, it is important to keep $W \geq 2$ GeV. With its luminosity and energy (in particular, with its future upgrades), CEBAF should be a good place to measure such power corrections, and to test the result shown in Eq. (12).

IV. MULTIPARTON CORRELATION IN A NUCLEUS

Multiparton correlation functions inside a large nucleus are extremely important and useful for understanding nuclear dependence in relativistic heavy ion collisions. Inside a large nucleus, multiple scattering, which is directly associated with the multiparton correlation functions, can take place within one nucleon or between different nucleons. Since the leading single scattering at high energy is always localized within one nucleon, it cannot generate any large dependence on nuclear size. Similarly, multiple scattering within one nucleon cannot provide much dependence on nuclear size either. Therefore, large dependence on nuclear size is a unique signal of multiple scattering between nucleons. Measurement of such anomalous dependence on the nuclear size for any physical observable will provide direct information on the multiparton correlation functions in a nucleus. However, in order to test QCD, and to extract the useful information on the multiparton correlation functions, it is important to identify the physical observables which depend only on a small number of correlation functions. Otherwise, the data will not be able to separate contributions from different multiparton correlation functions.

It was pointed out recently in Ref. [8] that at the leading order of α_s , it needs only one type of multiparton correlation function for both transverse momentum broadening of Drell-Yan pair and the jet broadening in DIS. It is a quark-gluon correlation function, and is defined as

$$\begin{aligned}
T_{qF}^A(x_B) &= \int \frac{dy^-}{2\pi} e^{ix_B p^+ y^-} \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-) \theta(-y_2^-) \\
&\times \langle P_A | F_\alpha^+(y_2^-) \bar{\psi}(0) \frac{\gamma^+}{2} \psi(y^-) F^{+\alpha}(y_1^-) | P_A \rangle, \quad (16)
\end{aligned}$$

with A the atomic weight. In terms of this correlation function, the jet broadening in DIS can be expressed as [8]

$$\Delta \langle p_T^2 \rangle = \frac{4\pi^2 \alpha_s}{3} \frac{\sum_q e_q^2 T_{qF}^A(x_B)}{\sum_q e_q^2 q^A(x_B)}, \quad (17)$$

where p_T is the transverse jet momentum in the photon-hadron frame in DIS. Measuring the jet momentum broadening, $\Delta\langle p_T^2 \rangle$, provides a direct measurement of the quark-gluon correlation functions inside a nucleus.

However, at the CEBAF energies, it is not possible to measure the jet cross section and corresponding jet momentum broadening in nuclear targets. But, it should be possible to measure the transverse momentum broadening of the leading pions (e.g., final-state pions with a relatively large momentum). Define the averaged transverse momentum of the pions of momentum ℓ as

$$\langle \ell_T^2 \rangle^{eA} = \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA \rightarrow \pi}}{dx_B dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dx_B dQ^2}}; \quad (18)$$

and define the nuclear broadening as

$$\Delta\langle \ell_T^2 \rangle = \langle \ell_T^2 \rangle^{eA} - \langle \ell_T^2 \rangle^{eN}. \quad (19)$$

We obtain [9] that

$$\Delta\langle \ell_T^2 \rangle = \frac{4\pi^2 \alpha_s}{3} \frac{\sum_q e_q^2 \int_{z_{\min}}^1 dz z^2 D_{q \rightarrow \pi}(z) T_{qF}^A(x_B)}{\sum_q e_q^2 q^A(x_B)}, \quad (20)$$

where $D_{q \rightarrow \pi}(z)$ are the quark-to-pion fragmentation functions, and z_{\min} depends on minimum pion momentum and is frame dependent. Since the quark-to-pion fragmentation functions are known, Eq. (20) provides the direct information on the quark-gluon correlation functions inside a nucleus. By measuring the transverse momentum broadening for both π^\pm and π^0 , and keep a reasonable large value of z_{\min} , we can extract the quark flavor dependence of the correlation functions.

V. CONCLUSIONS AND OUTLOOK

We present an analytical derivation of the leading twist-4 contributions to the DIS structure functions when x_B is large, and argue that CEBAF is a good place to test such contributions. Our derivation provides a direct linkage between the complete twist-4 contributions and the phenomenological parameterization used for fitting the existing data; and physical meanings of the parameters in the phenomenological parameterization. Besides the

similarities between the analytical result and the parameterization, there are also some differences. It is the difference that makes the measurements at CEBAF even more interesting.

We also demonstrated that measuring the transverse momentum broadening of the leading pion production at CEBAF can provide direct information on multiparton correlation functions in a large nucleus. Such information are extremely important for the multiple scattering and anomalous nuclear dependence in relativistic heavy ion collisions.

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