$N \to \Delta$ DVCS, exclusive DIS processes and skewed quark distributions in large N_c limit

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Abstract

We evaluate the amplitude of Bethe-Heitler process: $\gamma^* p \to \gamma \Delta^+$ - electron bremsstrahlung of photon accompanied by the excitation of Δ isobar. We show that this background is suppressed at small momentum transfer squared t to a proton relative to that for $\gamma^* p \to \gamma p$ and $\gamma^* p \to \gamma \Delta^+$ - DVCS processes. From experimental point of view, this means that $N \to N$ and $N \to \Delta$ skewed quark distributions (SQD's) might be measurable at small momentum transfer. Several implications and applications of the QCD factorization theorem for the processes $\gamma_L^* + p \to h_f + h_s$ are discussed where h_f - the particle produced along photon momentum $\vec{q}$ maybe either a meson or a baryon. We discuss also t dependence of DVCS and exclusive meson production as the practical criteria to distinguish between soft and hard regime. Basing on the large-N_c picture of the nucleon as a soliton of the effective chiral Lagrangian we derive relations between $N \to N$ and $N \to \Delta$ SQD's which can be used to estimate amplitude of $N \to \Delta$ DVCS .

Introduction

The aim of this presentation which summarizes talks given by the authors at the workshop is to discuss feasibility to separate DVCS from the background of inelastic processes related to the photon bremsstrahlung by electron and also to discuss some applications of the QCD factorization theorem for exclusive processes in DIS.

Recently, a new type of parton distributions [1, 2, 3, 4, 5] has attracted considerable interest, the so called skewed (non-forward, off–forward, non-diagonal) parton distributions (SPD's), which are generalizations simultaneously of the usual parton distributions, distribution amplitudes and the elastic nucleon form factors in the case of vacuum quantum numbers in t channel. At the same time there is a wide range of processes where one probes skewed densities which do not have a diagonal analog[5].

The SPD's are not accessible in standard inclusive measurements. They can, however, be measured in diffractive photoproduction of Z boson [1], in

diffractive electroproduction of vector mesons [2, 5, 6, 9],in deeply–virtual Compton scattering (DVCS) [3, 4, 9] and in hard exclusive electroproduction of mesons at moderate x_{Bj} [5, 7].

A quantitative description of these classes of processes requires not only knowledge of the perturbative evolution of the SPD's, but also non-perturbative information in the form of the SPD's at some initial normalization point. There were already model calculations of SPD's: in bag model [8] and in chiral quark-soliton model [10]. In the latter calculation strong dependence of flavor singlet SPD's on skewedness parameters was found. This dependence can considerably increase DVCS amplitude at moderately small x_{Bj} (see discussion below).

From experimental point of view the measurement of SPD's in DVCS is difficult because of strong Bethe-Heitler (BH) background at small momentum transfer t. To suppress the BH background one needs to increase the momentum transfer and hence beam energy what makes problematic such measurements at TJNAF. Alternative way to suppress the BH background is to study reaction like $\gamma^* p \to \gamma \Delta^+$, in this reaction the BH background is suppressed at small t relative to that in $\gamma^* p \to \gamma p$ because in the former case the e.m. $N \to \Delta$ transition form factor is always proportional to momentum transfer as a consequence of conservation of e.m. current. From theoretical point of view the measurements of $N \to \Delta$ SPD's can give additional insight into the structure of the nucleon and Δ.

$N \to \Delta$ BH process

The $N \to \Delta$ BH amplitude is expressed in terms of nucleon-Δ e.m. transition form factors. In our estimates we neglect G^*_{E2} and G^*_C transition form factors relative to G^*_M that, because the former numerically are very small at small momentum transfer we are interested in. We compute the ratio of amplitudes squared $R = |\mathcal{M}^{BH}_{p \to \Delta^+}|^2 / |\mathcal{M}^{BH}_{p \to p}|^2$ (suppression factor) in the Bjorken limit. In a kinematic domain $|t| \ll s$. Here "s" is the square of of invariant energy of ep collision. One can calculate leading term over powers of s and then to use conservation of em current to deduce generalization of he Weizsäcker–Williams method of equivalent photon. In this domain the suppression factor is given by simple formula:

$$R = -\frac{\mu^2_{N\Delta}(t - t_{min})}{4M^2_N}(1 + \frac{t}{4M^2_N}(\mu^2_p - 1)) , \qquad (1)$$

where t_{min} is minimal (in absolute value) momentum transfer in a reaction

$\gamma^* p \to \gamma \Delta^+$, which is given at large Q^2 by:

$$t_{min} = -\frac{x_{Bj}}{(1 - x_{Bj})}(x_{Bj}M_N^2 + M_\Delta^2 - M_N^2) \ , \tag{2}$$

$\mu_{N\Delta} = \sqrt{2/3}G_M^*(0) \approx (\mu_p - \mu_n)/\sqrt{2}$ is transitional magnetic moment, $\mu_{p,n}$ are magnetic moments of proton and neutron. From eq. (1) we see that the $N \to \Delta$ BH process is suppressed at small momentum transfer, numerically at $x_{Bj} = 0.1$ and $t = -0.15$ GeV^2 the suppression factor eq. (1) is about 0.18.

In the kinematic domain $x_{Bj}^2 M_N^2 \sim (M_\Delta - M_N)^2 \sim |t| \ll M_N^2$ we derive the following expression for the suppression factor:

$$R = \frac{\mu_{N\Delta}^2}{M_N^2} \frac{t - (M_\Delta - M_N)^2 - t(t - t_{min})\frac{1 - x_{Bj}}{2x_{Bj}^2 M_N^2}}{\mu_p^2 t + 4(1 - \frac{t}{4M_N^2}(\mu_p^2 - 1))(t + \frac{x_{Bj}^2 M_N^2}{1 - x_{Bj}})\frac{1 - x_{Bj}}{2x_{Bj}^2}} \ . \tag{3}$$

Evidently this expression is reduced to eq. (1) when $x_{Bj}^2 M_N^2 \ll |t|$. Numerically at $x_{Bj} = 0.1$ and $t = -0.1$ GeV^2 eq. (3) gives R of about $1/5$. Unfortunately, for the larger x_{Bj} the value of the factor R for $t = t_{min}$ gets larger: $R(x_{Bj} = 0.1) \approx 0.15, R(x_{Bj} = 0.2) \approx 0.35, R(x_{Bj} = 0.3) \approx 0.6$. However, the $1/Q$ effects neglected here should be studied.

QCD factorization theorem for exclusive DIS processes - some implications and applications

In [5] the QCD factorization theorem was proven for the process

$$\gamma^*(q) + p \to M(q + \Delta) + B'(p - \Delta) \tag{4}$$

at large Q^2, with t and $x = Q^2/2p \cdot q$ fixed. It asserts that the amplitude has the form

$$\sum_{i,j} \int_0^1 dz \int d\xi f_{i/p}(\xi, \xi - x; t, \mu) H_{ij}(Q^2\xi/x, Q^2, z, \mu) \phi_j(z, \mu)$$

$$+\text{power-suppressed corrections}, \tag{5}$$

where f is an SPD, ϕ is the light-front wave function of the meson, and H is a hard-scattering coefficient, usefully computable in powers of $\alpha_s(Q)$.

The proof of cancellation of the soft gluon interactions is intimately related to the fact that the meson arises from a quark-antiquark pair generated by the hard scattering. Thus the pair starts as a small-size configuration and only substantially later grows to a normal hadronic size, in the meson. This implies that the parton density is a standard parton density (apart from the skewed

nature of its definition). For example, no rescattering corrections are needed on a nuclear target, other than those that are implicit in the definition of universal parton densities, and that would equally appear in ordinary inclusive deep-inelastic scattering. These statements all apply to the leading power. This implies that the theorem is valid also for production of leading baryons

$$\gamma^*(q) + p \to B(q + \Delta) + M(p - \Delta) \tag{6}$$

and even leading antibaryons

$$\gamma^*(q) + p \to \bar{B}(q + \Delta) + B_2(p - \Delta) \tag{7}$$

Processes 6,7 will provide a unique information about multiparton correlations in nucleons. For example, the process 6 will allow to investigate what is probability for three quarks in a nucleon to come close together without collapsing the wave function into a three quark component - such probability would not be small in meson cloud models of the nucleon and in say MIT bag model. On the other hand if one would try to follow analogy with positronium, this probability would be strongly suppressed. At the same time the 7 would allow probability (presumably numerically very small) to have in a nucleon three antiquarks close together.

Note also that the process 6 could be used to produce a ρ-meson with a zero momentum if

$$q_0 = \frac{Q^2 + 2m_\rho m_N - m_\rho^2}{2(m_N - m_\rho)}. \tag{8}$$

Hence in the case of nuclear targets it allows to produce a ρ-meson with small momentum in the center of the nucleus. Therefore process 6 could be used for studing the medium modification of the properties of ρ-mesons. Advantage of this process as compared to low-energy processes is that effects of distortion of the two pion spectrum due threshold effects are less important in this case.

Though quantitative calculations of processes 6, 7 will take time, some qualitative predictions could be checked right away: the cross section of the process for fixed x, and large Q^2 should be proportional to the baryon elastic form factor. In particular it would be instructive to study the Q^2 dependence of the ratio of the the cross section of the process $\gamma^*(q) + p \to p + \pi^0$ and the square of the elastic proton form factor. If the color transparency suppresses the final state interaction between the fast moving nucleon and the residual meson state early enough one may expect that this ratio may reach the scaling limit in the region where higher twist contributions to the nucleon form factor are still large (It is worth emphasizing that the longitudinal distances involved in the final state interaction of the system flying along $\vec{q}$ with the residual system are much smaller in this case than in the case of $A(e, e'p)$ reaction,

so the expansion effects would be much less important in this case. Another interesting process is $\gamma^*(q) + p \to \Delta^{++} + \pi^-$ may allow to compare the wave functions of Δ-isobar and a nucleon in the way complementary to the $N \to \Delta$ transition processes.

Let us mention also two other applications of the factorization theorem:

(i) If the process 4 the leading meson is exotic - either a $q\bar{q}g$ or $q\bar{q}q\bar{q}$ state, the cross section of this process should decrease with Q^2 much faster than in the case of the $q\bar{q}$ mesons - by Q^4 in the $q\bar{q}g$ case, and Q^8 in the $q\bar{q}q\bar{q}$ case. Due to a mixing with $q\bar{q}$ states this fast drop of the cross section with Q^2 would be followed by a slower decrease but with much smaller absolute cross section than for the $q\bar{q}$ mesons. Thus the study of the Q^2 dependence of the production of candidates to the exotic mesons could help to check their quark-gluon content.

(ii) Study of the meson spectrum in the process $\gamma^* + p \to p + M$ may help to investigate the role of pions in the nucleon wave function. In a naive model of the nucleon with a pion cloud one would expect that production of a pion will dominate. However if more complicated nonlinear pion fields are important in the low Q^2 $q\bar{q}$ sea one may expect that higher masses would be at least as important. One can go a one step further and ask whether if one takes the valence three quarks out of the nucleon - could be the residual system couple strongly enough to the gluonium states, providing another avenue for looking for exotic meson states.

How to distinguish experimentally between hard and soft QCD regimes?

Here we want to explain important advantage of identification of hard QCD physics in DVCS and more generally in hard exclusive processes as compared to that in e.m. form factors of hadrons. In both processes there is competition between soft QCD physics (end point contribution=Feynman mechanism) and hard QCD physics [14, 15]. Serious problem for the investigation of e.m. form factors of hadrons is absence of unambiguous criteria to distinguish between both contributions [16]. On the contrary the dependence of hard exclusive processes on momentum transfered to proton can be used as unambiguous criteria of dominance of soft or hard physics in the process.

Cross section of two body process can be parameterized at large energies as $d\sigma(\gamma^* + p \to M + T)/dt = A\exp(Bt)$. Here M can be photon,vector or pseudo-scalar meson. The slope B can be parameterized as $B = B_0(Q^2) + 2\alpha'(Q^2)\ln(s/\mu^2)$. Common wisdom based on the success of Regge pole hypothesis in the description various two body high energy processes is that for the soft QCD processes B should be independent on Q but it should depend on the energy s. We presented above the parameterization of cross section

which is valid for the exchange by Regge pole. On the contrary for hard exclusive processes scale of dependence of hard vertex on t is given by Q. Thus $B_0 \to B_N$ in the limit of large Q. Here $B > B_N$ and B_N should be the same for all hard processes with the same quantum numbers in the crossed channel. [2]. Moreover $\alpha'(Q^2) \to 0$ also in the limit of large Q. This is because QCD evolution for skewed parton distributions prevents Gribov diffusion of small gluon wave package to large impact parameters. Predicted dependence of B for the hard exclusive processes on Q^2 agrees with HERA data on diffractive production of vector mesons for large Q^2 [17]. The difference between soft and hard processes should be most striking for the processes with non-vacuum quantum numbers in the crossed channel like excitation of Δ. Really in the case of usual Regge pole trajectories; $\alpha' = 1 GeV^{-2}$ and dominance of Regge pole exchange should reveal itself at energies of projectile of few GeV - famous Dolen-Horn-Schmidt duality. Thus criteria of dominance of hard QCD is decrease of the slope B with increase of Q^2 can be observed at moderately large s. So this physics could be studied already at TJNAF and HERMES energies. (physics for TJNAF,HERMES and Compass). The limiting value of $B(Q \to \infty)$ should be significantly smaller than for $Q = 0$ and independent of energy and Q^2.

N → Δ skewed quark distributions

The soft part of the DVCS $N \to \Delta$ amplitude is parameterized generically in terms of eight $(\times N_f)$ $N \to \Delta$ skewed quark distributions (SQD's) (_cf._ four$(\times N_f)$ for $N \to N$ SQD's) [the detailed expression will be given elsewhere [12]].

In order to relate $N \to \Delta$ SQD's to $N \to N$ ones we shall use the large N_c picture of the baryons. In this picture the nucleon and Δ are different rotational states of the same object–the "classical" or "generalized" nucleon. Therefore the static properties of nucleon and Δ can be related to each other, a number of such relations were derived in the past (see [11]) and it turns out that such relations work very well. Let us give a few examples of such relations:

$$\frac{\mu_{N\Delta}}{\mu_p - \mu_n} = \frac{1}{\sqrt{2}} \ (\text{expt. } 0.71 \pm 0.01), \quad \frac{g_{\pi N \Delta}}{g_{\pi N N}} = \frac{3}{2} \ (\text{expt. } 1.5 \pm 0.12). \quad (9)$$

Such kind of relations is independent of particular dynamical realization of idea baryon as chiral soliton.

Below we sketch an idea how to relate $N \to \Delta$ SQD's to $N \to N$ ones, detailed account of the calculations will be given elsewhere [12]. First we write the matrix element of bilocal quark operator on the light cone between soliton ("generalized" nucleon) states. [Technique how to calculate such matrix

elements was developed in [13], we refer the reader to this paper for details]
These matrix elements are expressed in terms of operators in collective coordinate space. The projection on a baryon state with given spin and isospin components is obtained by integrating over all spin-isospin rotations, R [11],

$$\langle S' = T', S_3', T_3' | \ldots | S = T, S_3, T_3 \rangle = \int dR \, \phi^{*\,S'=T'}_{S_3'T_3'}(R) \ldots \phi^{S=T}_{S_3 T_3}(R) \quad (10)$$

Here $\phi^{S=T}_{S_3 T_3}(R)$ is the rotational wave function of the baryon given by the Wigner finite-rotation matrix [11]:

$$\phi^{S=T}_{S_3 T_3}(R) = \sqrt{2S+1}(-1)^{T+T_3} D^{S=T}_{-T_3, S_3}(R). \quad (11)$$

Analogously, the projection on a baryon state with given momentum $\mathbf{P}$ is obtained by integrating over all shifts, $\mathbf{X}$, of the soliton,

$$\langle \mathbf{P}' | \ldots | \mathbf{P} \rangle = \int d^3 \mathbf{X} \, e^{i(\mathbf{P}' - \mathbf{P}) \cdot \mathbf{X}} \ldots \quad (12)$$

It can be shown [12] that in the leading order of $1/N_c$ expansion the soliton (the "classical" or "generalized" nucleon) is characterized (for $N_f = 2$) by 4 SQD's (7 in the next to leading order). Because both $N \to N$ and $N \to \Delta$ SQD's are expressible in terms of the *same* SQD's of the "generalized" nucleon there are a number of relations between them. Because of lack of space we give here the simplest relation between DVCS amplitudes squared for $N \to N$ and $N \to \Delta$ transitions in the leading order of $1/N_c$ expansion:

$$\left| \mathcal{M}^{DVCS}_{p \to \Delta^+} \right|^2 = \frac{1}{2} \left| \mathcal{M}^{DVCS}_{p \to p} - \mathcal{M}^{DVCS}_{n \to n} \right|^2 \left(1 + O(\frac{1}{N_c}) \right). \quad (13)$$

A few comments are in order here. First, the above relation is hold for Bjorken x_{Bj} of order of $1/N_c$, *i.e.* in valence region, it is modified at x_{Bj} close to unity (see discussion in [13]). Second, the $1/N_c$ corrections (denoted $O(1/N_c)$ in eq. (13)) are also expressible solely in terms of $N \to N$ SQD's and hence can be estimated numerically, the corresponding results are in preparation [12].

The simplest estimate of $N \to \Delta$ DVCS amplitude squared eq. (13) shows that the $N \to \Delta$ DVCS amplitude is comparable with $N \to N$ that. The detailed estimate will be given elsewhere [12].

Concluding remarks

We obtained that the BH background to the $N \to \Delta$ DVCS is suppressed at small momentum transfer, the BH cross section behaves $\sim t^0$ at small

momentum transfer, whereas cross section of the $N \to N$ BH process behaves like $\sim 1/t$. The DVCS cross sections in both cases are of the same order. This makes the measurements of the $N \to \Delta$ DVCS favorable at (upgraded) TJNAF energies. Such measurements can give new information of nucleon and Δ inner structures. Interesting to note that $N \to \Delta$ DVCS at small momentum transfer is especially sensitive to the helicity dependent SPD's.

Another interesting possibility is to measure the reaction $\gamma^* N \to \gamma(k)\pi N$ ($k = 1, 2, \ldots$) with invariant mass of the $(k)\pi N$ system below and in resonance region. It is correct that BH background in this case is suppressed at small t, The process $\gamma^* N \to \gamma(k)\pi N$ is described by the generalized SPD's, which additionally to x, skewedness and t depend on invariant mass of $(k)\pi N$ system and on the distribution of longitudinal momentum between the final hadrons. These generalized SPD's can be calculated in chiral quark soliton model. Anyway, study of say $N \to \pi N$ SPD's is required to estimate non-resonant background to $N \to \Delta$ DVCS[1].

Let us also note that the estimates of the DVCS cross section made in [18] were based on oversimplified models of SPD's. In particular, the dependence of the SPD's on skewedness parameter was neglected and the SPD's which correspond to helicity flip amplitudes were neglected. The authors of ref. [18] correctly argue that the contribution of the latter SQD's is small at small t. But contribution of the pion pole of the type $\sim g_A^2 t/(|t| + m_\pi^2)^2$ may appear not small at $|t| \sim m_\pi^2$.

In our opinion the simulations of the DVCS with more realistic models for SPD's are needed.

Also one should try to design experiments which would be able to study a broad range of processes 4,6,7 including channels with strangeness. This would allow to expand immensely our understanding of the short-range hadron structure.

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