Longitudinal and transverse structure functions of proton and deuteron at large x

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Abstract

Higher-twist effects in the low-order moments of the longitudinal and transverse structure functions of proton and deuteron have been analyzed using available phenomenological fits of existing data in the Q^2 range between 1 and 20 $(GeV/c)^2$. Both twist-4 and twist-6 contributions have been determined adopting the Natchmann definition of moments, which allows to disentangle properly target-mass effects. The extraction of the matrix elements of the relevant twist-4 operators, describing quark-quark and quark-gluon correlations, is carried out in case of the second moment. The need of transverse data with better quality for $x \gtrsim 0.5$ and $Q^2 \lesssim 10$ $(GeV/c)^2$ as well as more precise and systematic determinations of the L/T separation make $JLab$ @ 12 GeV a good place to improve our understanding of the non-perturbative structure of hadrons.

1 INTRODUCTION.

The experimental investigation of deep-inelastic lepton-nucleon scattering has provided a wealth of information on the occurrence of Bjorken scaling and its violations, giving a decisive support to the rise of the parton model and its QCD-improved version, which properly describe the logarithmic violations to scaling in the asymptotic region. However, in the pre-asymptotic region the full dependence of the nucleon response on the squared four-momentum transfer, Q^2, is affected also by power-type corrections, which originate from non-perturbative physics and can be analyzed in the framework provided by the Operator Product Expansion (OPE). The logarithmic scale dependence is therefore related to the so-called leading twist operators, which in the parton language are one-body operators whose matrix elements yield the contribution of the individual partons to the nucleon response. On the contrary, power-type corrections are related to higher-twist operators measuring the relevance of correlations among partons [1].

In case of unpolarized inelastic electron scattering the nucleon response is described by two independent quantities: the transverse $F_2(x, Q^2)$ and the longitudinal $F_L(x, Q^2)$ structure functions, the latter being related to the ratio of the longitudinal to transverse cross sections, $R_{L/T}(x, Q^2)$, by $F_L(x, Q^2) = F_2(x, Q^2)\ (1 + 4M^2x^2/Q^2)R_{L/T}(x, Q^2)/[1 + R_{L/T}(x, Q^2)]$, where $x \equiv Q^2/2M\nu$ is the Bjorken variable, M is the nucleon mass and ν is the energy transfer in the nucleon rest frame. Systematic measurements [2] of the transverse function $F_2(x, Q^2)$ for proton and deuteron targets have been carried out in the kinematical range $10^{-4} \lesssim x \lesssim 1$ and for Q^2 values up to several hundreds of $(GeV/c)^2$, while data on the ratio $R_{L/T}(x, Q^2)$ are available for $0.002 \lesssim x \lesssim 0.8$ and $0.5 \lesssim Q^2 (GeV/c)^2 \lesssim 70$, though they are still fluctuating and affected by large errors. Consequently, phenomenological fits for both $F_2(x, Q^2)$ and $F_L(x, Q^2)$ are available, but for the latter quantity the interpolation formulae greatly suffer for very weak constraints. The analysis of the world data set [2] has allowed to extract the parton densities in the nucleon, including their QCD-predicted logarithmic Q^2 evolution, as well as to signal the presence of power-type scaling violations at *large* x ($\gtrsim 0.7$) and *low* Q^2 ($\lesssim 10$ $(GeV/c)^2$). The analysis of these kinematical regions, where higher-twist effects are important, represents the aim of the present contribution. A more detailed version of our work will be available soon in [4].

2 TWIST ANALYSIS.

An important and effective tool for the theoretical investigation of the complete Q^2 dependence of hadron structure functions is the OPE, which leads to the well-known twist expansion for the moments of the structure functions. In our analysis we do not use the Cornwall-Norton definition of the moments, since target-mass corrections, i.e. terms containing powers of M^2/Q^2, would contribute. Instead of that we will adopt the Natchmann definition [3]:

$$M_n^{(T)}(Q^2) \equiv \int_0^1 dx \frac{\xi^{n+1}}{x^3} F_2(x,Q^2) \frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)} \tag{1}$$

$$M_n^{(L)}(Q^2) \equiv \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left[F_L(x,Q^2) + \frac{4M^2 x}{Q^2} F_2(x,Q^2) \frac{(n+1)\xi - 2(n+2)x}{(n+2)(n+3)} \right] \tag{2}$$

where $n \geq 2$, $r \equiv \sqrt{1+4M^2x^2/Q^2}$ and $\xi \equiv 2x/(1+r)$ is the Natchmann variable. Using the *experimental* $F_2(x,Q^2)$ and $F_L(x,Q^2)$ in Eqs. (1-2), target-mass effects are canceled out and therefore the twist expansions of the experimental $M_n^{(T)}(Q^2)$ and $M_n^{(L)}(Q^2)$ contain only *dynamical* twists, namely

$$M_n^{L(T)}(Q^2) = \sum_{\tau=2}^{\infty} C_{n,\tau}^{L(T)}(Q^2/\mu^2)\ A_{n,\tau}^{L(T)}(\mu^2)\ (\mu^2/Q^2)^{\frac{\tau-2}{2}} \tag{3}$$

where μ is the renormalization scale, $C_{n,\tau}^{L(T)}(Q^2/\mu^2)$ is a Wilson coefficient calculable within perturbative QCD and $A_{n,\tau}^{L(T)}(\mu^2)$ corresponds to the matrix elements of operators of twist τ and spin n. In our analysis the expansion (3) is simplified into

$$M_n^{L(T)}(Q^2) = A_n^{L(T)}(Q^2) + a_n^{(4)}[L(T)] \frac{\mu^2}{Q^2} \left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right)^{\gamma_n^{(4)}[L(T)]} + a_n^{(6)}[L(T)] \frac{\mu^4}{Q^4} \left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right)^{\gamma_n^{(6)}[L(T)]} \tag{4}$$

where $\alpha_s(Q^2)$ is the running coupling constant and $A_n^{L(T)}(Q^2)$ is the leading-twist contribution, whose Q^2 dependence is calculated according to the $pQCD$ predictions at NLO. In Eq. (4) the last two terms in the r.h.s. are simplified parametrizations of the twist-4 and twist-6 contributions, respectively, as suggested in Ref. [5]; the quantities $a_4^n[L(T)]$ ($a_6^n[L(T)]$) and $\gamma_4^n[L(T)]$ ($\gamma_6^n[L(T)]$) represent the effective magnitude and anomalous dimension of twist-4 (twist-6) operators.

3 MAIN RESULTS.

Equation (4) has been applied to the fit of the Q^2 behavior of the *experimental* moments $M_n^{L(T)}(Q^2)$ in the range $1 \lesssim Q^2(GeV/c)^2 \lesssim 20$. In order to evaluate the r.h.s. of Eqs. (1-2) available phenomenological fits based on the data of Ref. [2] have been used and the elastic peak contributions have been added, as it is required by the inclusive nature of the OPE. In case of the deuteron, the nucleon elastic peak leads to the quasi-elastic contribution to the inclusive cross section, which has been evaluated through a convolution approach checked against available $SLAC$ data (see [4] for details). In case of the analysis of the transverse moments, besides the free parameters $a_n^{(\tau)}$ and $\gamma_n^{(\tau)}$ appearing in the twist-4 and twist-6 terms of Eq. (4), also the magnitude of the leading term is simultaneously determined from the fit of our pseudo-data. On the contrary, the parton densities of Ref. [6] have been adopted for evaluating $A_n^L(Q^2)$ (see again [4] for details). Thanks to the decoupling of the singlet-quark and gluon operators at large x, the non-singlet evolution of the leading twist can be safely applied for the analysis of the moments with $n \geq 4$. In our work we have adopted a renormalization scale $\mu = 1\ GeV$ as in [5].

The main results of our analysis of the transverse moments for both proton and deuteron targets can be summarized as follows: i) the simplified twist expansion (4), containing up to the twist-6 term, is able to reproduce the Q^2 dependence of the transverse moments starting from $Q^2 \simeq 1\ (GeV/c)^2$; ii) the second moment $M_2^T(Q^2)$ is only slightly affected by the twist-4 and almost unaffected by the twist-6 (see Fig. 1(a)); iii) on the contrary, both twist-4 and twist-6 significantly contribute to the moments of order $n \geq 4$ (see Fig. 1(b)), in accord with the presence of higher-twist effects at large x; iv) the signs of the twist-4 and twist-6 contributions turn out to be opposite.

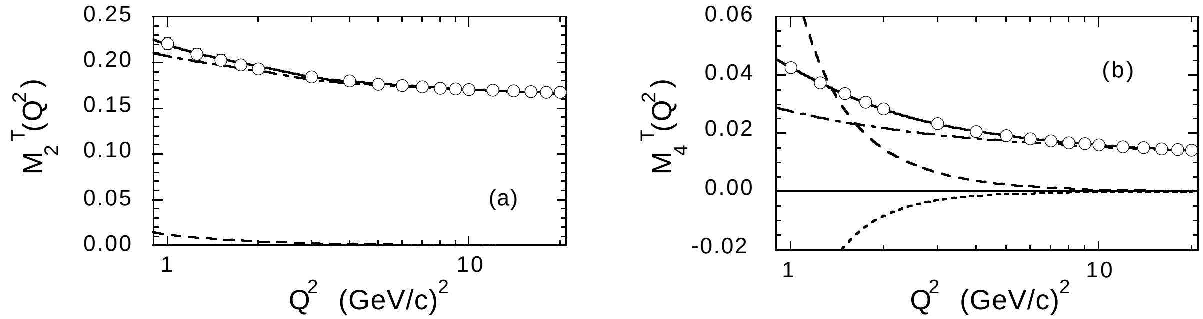

Figure 1. Second (a) and fourth (b) moments of the proton transverse structure function versus Q^2. Open dots: pseudo-data calculated by Eq. (1) using available phenomenological fits of existing data [2] on $F_2(x, Q^2)$; the error bars are obtained using the quoted uncertainties of the phenomenological fits. Solid lines: result of our twist analysis based on Eq. (4); dashed-dotted, dashed and dotted lines: twist-2, twist-4 and twist-6 contributions, respectively.

Basing on naive counting arguments, one can argue that the twist expansion (4) of the transverse moments for $Q^2 \sim \mu^2$ can be rewritten as: $M_n^T(\mu^2) = A_n^T(\mu^2)[1 + n(\mu_n^{(4)}/\mu)^2 - n^2(\mu_n^{(6)}/\mu)^4]$, with $\mu_n^{(\tau)}$ approximately independent of n for $n \gtrsim 4$. Thus, one gets

$$\mu_n^{(4)} = \mu \sqrt{\frac{a_n^{(4)}[T]}{n A_n^T(\mu^2)}}, \qquad \mu_n^{(6)} = \mu \left[\frac{|a_n^{(6)}[T]|}{n^2 A_n^T(\mu^2)} \right]^{1/4}. \tag{5}$$

Our results for $\mu_n^{(\tau)}$ are collected in Fig. 2, where it can clearly be seen that the mass scales of the twist-4 and twist-6 terms of our analysis are $\mu_n^{(4)} \simeq \mu^{(4)} \simeq 1\ GeV$ and $\mu_n^{(6)} \simeq \mu^{(6)} \simeq 0.6\ GeV$. The value obtained for $\mu^{(4)}$ is significantly higher than the naive expectation $\mu^{(4)} \simeq \sqrt{< k_\perp^2 >} \simeq 0.3\ GeV$ [7] as well as higher than the result of other twist-4 analyses (see [5]).

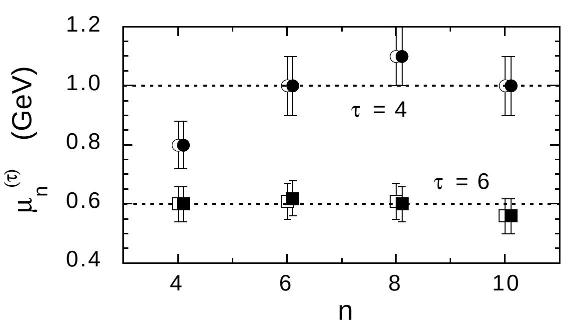

Figure 2. The mass scale $\mu_n^{(\tau)}$(Eq. (5)) of the twist-4 and twist-6 terms of our twist analysis (4). Open and full markers correspond to the proton and deuteron case, while dots and squares are our results for the twist-4 and twist-6, respectively.

In case of the longitudinal channel the uncertainties in the calculation of the moments are remarkably larger than those of the transverse ones. The effects of the higher-twists are still dominant in the second moment $M_2^L(Q^2)$ up to Q^2 of several $(GeV/c)^2$ (see Fig. 3(a)). Note that the moments with $n \geq 4$ can be reproduced by considering the leading twist plus a twist-4 term only (see Fig. 3(b)).

The main goal of the investigation of higher-twist effects is to disentangle the separate contributions of the various operators of a given twist yielding the relevant multiparton correlations. This is not an easy

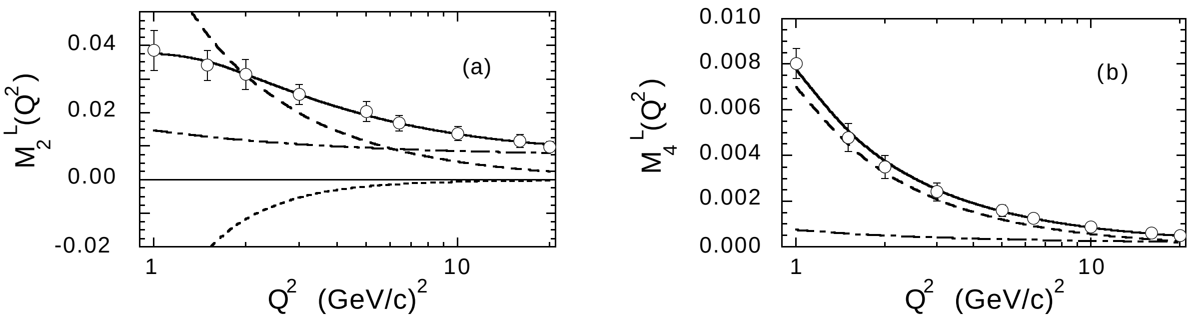

Figure 3. The same as in Fig. 1, but in case of the proton longitudinal structure function.

task, due to the contributions of many operators for any given twist (see [1,8]). Following Ref. [1], there are seven twist-4 operators contributing to the second moment: three in the non-singlet channel and four in the singlet one; the explicit expressions of these operators can be read off from [1]. In the non-singlet case it is possible to write down three independent equations using, besides $M_2^T(Q^2)$ and $M_2^L(Q^2)$, the second moment of the structure function $F_3(x, Q^2)$, which can be determined in neutrino and antineutrino scattering experiments, like the recent measurement performed by the $CCFR$ collaboration [9] at FermiLab. The neutrino data have been analyzed at $NNLO$ in Ref. [10], obtaining a determination of the twist-4 contribution. Using all these experimental results and adopting the notation of Ref. [1], we have got: $A^{NS} = -9.0 \pm 4.5$ and $B^{NS} = -2.0 \pm 0.4$ for the quark-gluon correlation operators, and $C^{NS} = 5.2 \pm 2.8$ for the quark-quark correlation operator. In the singlet case the neutrino data on $F_3(x, Q^2)$ cannot help in determining the various twist-4 operators; assuming that the quark-quark correlation matrix element C^S is the same in neutrino and electron experiments, we have got the following constraints: $A^S + 6B^S = 12.5 \pm 1.8$ and $8C^S + 5A^S - 2B^S = 0$.

Finally, we point out that transverse data with better quality are still needed for $x \gtrsim 0.5$ and $Q^2 \lesssim 10\ (GeV/c)^2$ as well as more precise and systematic determinations of the L/T separation are required; for these reasons an electron facility, like $JLab$ @ 12 GeV, is a good place to investigate multiparton correlations in the hadron structure.

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